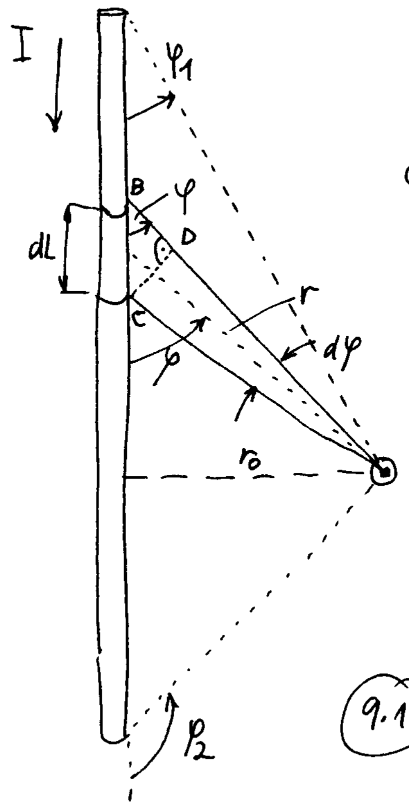
9.1 / 9.2

Na riešenie použijeme Biotov-Savartov zákon (formulovaný r. 1820) a zovšeobecnený Laplasom:

$$d\vec{B} = \frac{\mu_0 I}{4\pi\, r^3}\left(d\vec{L} \times \vec{r}\right)$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\,\frac{I dL \cdot \sin\varphi}{r^2}$$



$$\frac{r_0}{r} = \sin\varphi \;,\quad dl = \frac{CD}{\sin\varphi} = \frac{r\,d\varphi}{\sin\varphi} = \frac{r_0}{\sin^2\varphi}\,d\varphi$$

$CD = r\,dL$

$$dB = \frac{\mu_0 I}{4\pi}\cdot\frac{r_0\,d\varphi}{\sin^2\varphi}\cdot\frac{\sin\varphi}{\frac{r_0^2}{\sin^2\varphi}} = \frac{\mu_0 I}{4\pi}\cdot\frac{\sin\varphi\,d\varphi}{r_0}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\varphi_1}^{\varphi_2}\frac{\sin\varphi\,d\varphi}{r_0} = \frac{\mu_0 I}{4\pi r_0}\left[-\cos\varphi\right]_{\varphi_1}^{\varphi_2} =$$

$$= \frac{\mu_0 I}{4\pi r_0}\left(\cos\varphi_1 - \varphi_2\right)$$

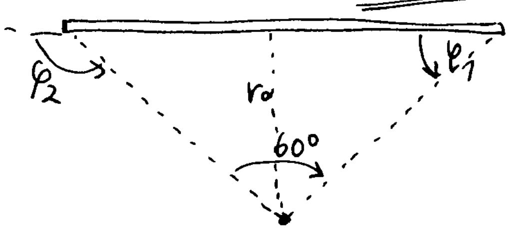
9.1 Pre veľmi dlhý priamy vodič

$$\varphi_1 \to 0 \;,\quad \varphi_2 \to \pi$$

$$B = \frac{\mu_0 I}{4\pi r_0}\left[1-(-1)\right] = \frac{\mu_0 I}{2\pi r_0} = \frac{4\pi\cdot 10^{-7}\cdot 5}{2\cdot\pi\cdot 5\cdot 10^{-2}} =$$

$$= \underline{2\cdot 10^{-5}\,T}\;,\quad H = B/\mu_0 = \underline{15{,}915\,494\ A\cdot m^{-1}}$$

9.2



$$\varphi_1 + (\pi - \varphi_2) + \frac{\pi}{3} = \pi$$

$$\varphi_1 - \varphi_2 = -\frac{\pi}{3}$$

$$\varphi_2 = \frac{\pi}{3} + \varphi_1$$

$$2\varphi_1 + \frac{\pi}{3} = \pi$$

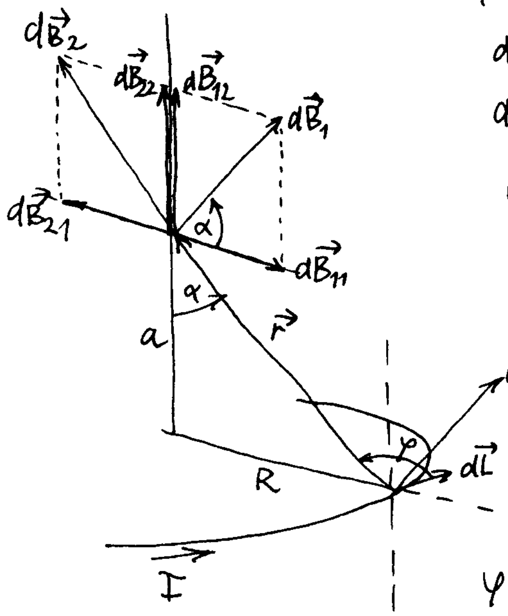
$$\varphi_1 = \frac{2}{6}\pi = \frac{\pi}{3} = \underline{60°}$$

$$\varphi_2 = \frac{\pi}{3} + \frac{\pi}{3} = \underline{120°}$$

$$B = \mu H$$

$$H = \frac{1}{4\pi r_0}\left(\cos\varphi_1 - \cos\varphi_2\right) = \frac{10}{4\pi\cdot 5\cdot 10^{-2}}\left[\frac{1}{2} - \left(-\frac{1}{2}\right)\right] = \frac{100}{2\pi} = \underline{15{,}91549\ A\cdot m^{-1}}$$

9.6



$$|d\vec{B}_1| = |d\vec{B}_2| = |d\vec{B}|$$

$$d\vec{B}_{11} = -d\vec{B}_{21}$$

$$d\vec{B}_{12} = d\vec{B}_{22}$$

$$|d\vec{B}_{12}| = |d\vec{B}_{22}| = dB \cdot \sin\alpha$$

$$\boxed{B = \oint dB \cdot \sin\alpha}$$

$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{dL \cdot \sin\varphi}{r^2}$$

$$r^2 = a^2 + R^2$$

$$\sin\alpha = \frac{R}{r} = \frac{R}{\sqrt{a^2+R^2}}$$

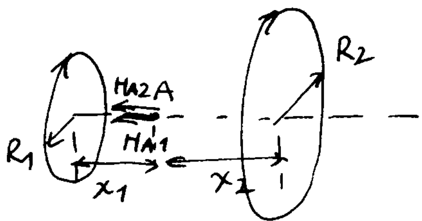
$$\varphi = \frac{\pi}{2}, \quad \sin\varphi = 1$$

$$B = \oint \frac{\mu_0 I}{4\pi} \cdot \frac{dL \cdot R}{(a^2+R^2)\sqrt{a^2+R^2}} = \frac{\mu_0 I R}{4\pi (a^2+R^2)^{3/2}} \underbrace{\oint_{(L)} dL}_{2\pi R} =$$

$$= \underline{\underline{\frac{\mu_0 I}{2} \cdot \frac{R^2}{(a^2+R^2)^{3/2}}}} = \frac{4\pi \cdot 10^{-7} \cdot 2 \cdot (10 \cdot 10^{-2})^2}{2\left[(10 \cdot 10^{-2})^2 + (10 \cdot 10^{-2})^2\right]^{3/2}} =$$

$$= \frac{4\pi \cdot 10^{-7} \cdot 100 \cdot 10^{-4}}{(2 \cdot 100 \cdot 10^{-4})^{3/2}} = \frac{4\pi}{\sqrt{8}} \cdot 10^{-6} = \underline{\underline{4{,}442\,883 \cdot 10^{-6}\,T}}$$

9.7



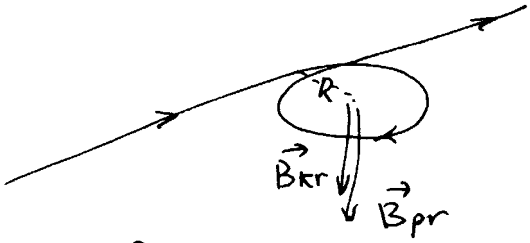
$$\vec{H}_A = \vec{H}_{A1} + \vec{H}_{A2} \Rightarrow |\vec{H}_A| = H_{A1} + H_{A2}$$

Z výsledku úlohy 9.6 plyne: $H_{A(1,2)} = \dfrac{I_{(1,2)}}{2} \dfrac{R^2_{(1,2)}}{\left[x^2_{(1,2)} + R^2_{(1,2)}\right]^{3/2}}$

$$H_A = \frac{2 \cdot (10 \cdot 10^{-2})^2}{2\left[(5 \cdot 10^{-2})^2 + (10 \cdot 10^{-2})^2\right]^{3/2}} + \frac{5 \cdot (15 \cdot 10^{-2})^2}{2\left[(10 \cdot 10^{-2})^2 + (15 \cdot 10^{-2})^2\right]^{3/2}} =$$

$$= 7{,}155\,420 + 9{,}600\,579 = 16{,}755\,999\ A \cdot m^{-1} \approx 16{,}8\,A \cdot m^{-1}$$

9.10



$$\vec{B} = \vec{B}_{kr} + \vec{B}_{pr} \quad \Longrightarrow \quad |\vec{B}| = B_{kr} + B_{pr}$$

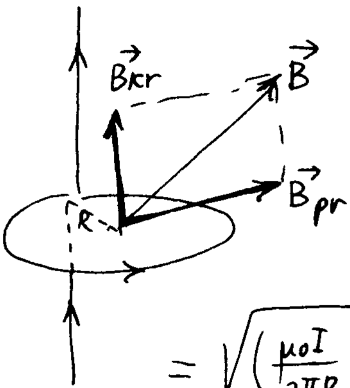
Z výsledkov úloh 9.1 a 9.6 môžeme napísať:

$$a = 0 \,, \quad r_0 = R$$

$$|\vec{B}| = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1 + \frac{1}{\pi}\right) =$$

$$= \frac{4\pi \cdot 10^{-7} \cdot 10}{2 \cdot 4{,}28 \cdot 10^{-2}}\left(1 + \frac{1}{\pi}\right) = \underline{\underline{1{,}935\,323 \cdot 10^{-4}\,\mathrm{T}}}$$

9.11



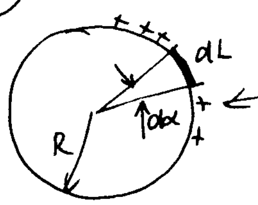
$$\vec{B} = \vec{B}_{pr} + \vec{B}_{kr}$$

$$|\vec{B}| = \sqrt{B_{pr}^2 + B_{kr}^2} =$$

$$= \sqrt{\left(\frac{\mu_0 I}{2\pi R}\right)^2 + \left(\frac{\mu_0 I}{2R}\right)^2} = \frac{\mu_0 I}{2R}\sqrt{\frac{1}{\pi^2} + 1} =$$

$$= \frac{4\pi \cdot 10^{-7} \cdot 10}{2 \cdot 4{,}28 \cdot 10^{-2}}\sqrt{\frac{1}{\pi^2} + 1} = \underline{\underline{1{,}540\,611 \cdot 10^{-4}\,\mathrm{T}}}$$

9.12



Rovnomerne rozložený náboj na obvode kotúča

$$I = \frac{dq}{dt} \,, \quad dq = \tau\, dL = \tau R \cdot d\alpha$$

$$I = \tau R \frac{d\alpha}{dt} = \tau R \omega = \frac{q}{2\pi R} \cdot R \cdot 2\pi f = q f$$

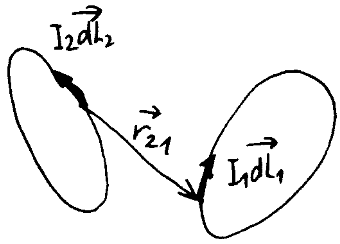
Z výsledku úlohy 9.6 vyplýva $(a = 0)$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q}{2R} \cdot f = \frac{4\pi \cdot 10^{-7} \cdot 10^{-8} \cdot 100}{2 \cdot 10 \cdot 10^{-2}} = \underline{\underline{6{,}283\,185 \cdot 10^{-12}\,\mathrm{T}}}$$

$$H = \frac{B}{\mu_0} = 4{,}999\,999 \cdot 10^{-6}\,\mathrm{A \cdot m^{-1}} \approx 5 \cdot 10^{-6}\,\mathrm{A \cdot m^{-1}}$$

9.13

Na riešenie použijeme Ampérov zákon vo všeobecnom tvare:

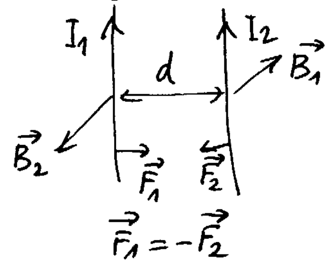


$$\vec{F}_{21} = \frac{\mu_0 I_1 I_2}{4\pi} \oint d\vec{l}_1 \times \oint \frac{d\vec{l}_2 \times \vec{r}}{r^3}$$

Po úprave pre prípad dvoch rovnobežných vodičov

$$\boxed{|\vec{F}| = \frac{\mu_0 I_1 I_2}{2\pi d} \cdot L}$$

Na každý úsek dĺžky L jedného vodiča pôsobí od celého druhého vodiča táto sila.



a/ $dF = I \cdot B \cdot dL = I \cdot \frac{B_{kr}}{2} \cdot dL$

$$\frac{dF}{dL} = I \cdot \frac{\mu_0 I}{2 \cdot 2R} = \frac{\mu_0 I^2}{4R} = \frac{4\pi \cdot 10^{-7} \cdot 8^2}{4 \cdot 10 \cdot 10^{-2}} = \underline{\underline{2{,}010\,610 \cdot 10^{-4}\ N \cdot m^{-1}}}$$

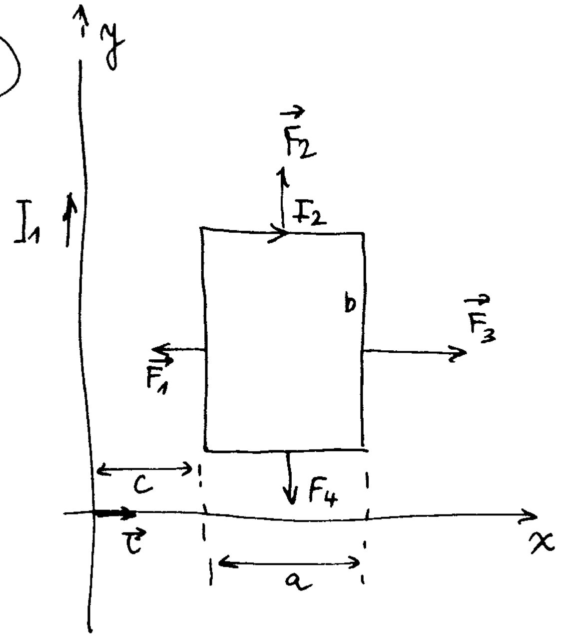
b/ $\frac{dF}{dL} = 2 \cdot B_{pr} \cdot I$

Z výsledku $\frac{9.1}{9.2}$ pre $\varphi_1 = \frac{\pi}{2}$ a $\varphi_2 = \pi$

$$B_{pr} = \frac{\mu_0 I}{4\pi \left(\frac{L}{2}\right)} \cdot \left[0 - (-1)\right] = \frac{\mu_0 I}{2\pi L}$$

$$\frac{dF}{dL} = 2 \cdot \frac{\mu_0 I}{2\pi L} \cdot I = 2 \frac{4\pi \cdot 10^{-7} \cdot 8^2}{2\pi \cdot 20 \cdot 10^{-2}} = \underline{\underline{1{,}28 \cdot 10^{-4}\ N \cdot m^{-1}}}$$

9.16



$$\vec{F}_2 = -\vec{F}_4$$

$$\vec{F}_2 + \vec{F}_4 = 0$$

Podľa Flemingovho pravidla ľavej ruky určíme smer sily $\vec{F}_1$ a sily $\vec{F}_3$. Pre výslednú silu pôsobiacu na vodič tvaru obdĺžnika platí $F = F_1 - F_3$

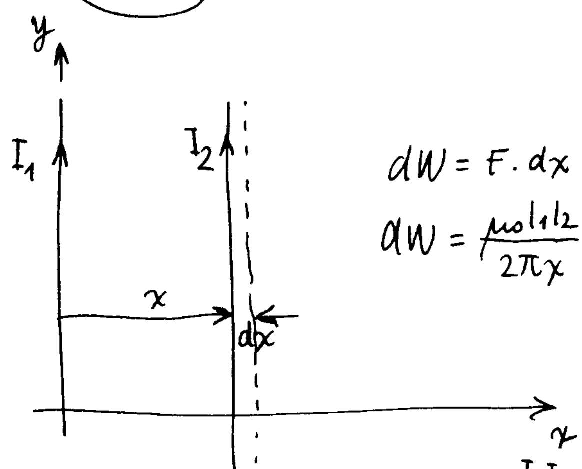
$$F = \frac{\mu_0 I_1}{2\pi c} I_2 \cdot b - \frac{\mu_0 I_1}{2\pi(a+c)} I_2 \cdot b = \frac{\mu_0 I_1 I_2}{2\pi} b \left(\frac{1}{c} - \frac{1}{a+c}\right)$$

Vynásobením rovnice jednotkovým vektorom $\vec{\tau}$ dostávame

$$F \cdot \vec{\tau} = \frac{\mu_0 I_1 I_2}{2\pi} b \left(\frac{1}{c} - \frac{1}{a+c}\right) \vec{\tau}$$

$$\vec{F} = \frac{\mu_0 I_1 I_2}{2\pi} b \left(\frac{1}{c} - \frac{1}{a+c}\right) \vec{\tau}$$

9.17



$$dW = F \cdot dx$$

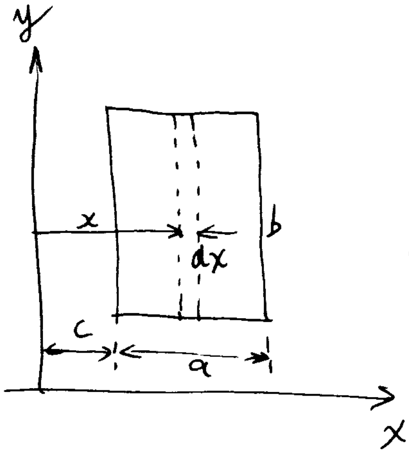
$$dW = \frac{\mu_0 I_1 I_2}{2\pi x} \cdot L \cdot dx$$

$$W = \frac{\mu_0 I_1 I_2}{2\pi} L \int_d^{3d} \frac{dx}{x} = \frac{\mu_0 I_1 I_2}{2\pi} \cdot L \cdot \ln 3$$

$$\frac{W}{L} = \frac{\mu_0 I_1 I_2}{2\pi} \cdot \ln 3 = \frac{4\pi \cdot 10^{-7} \cdot 40 \cdot 30}{2\pi} \ln 3 =$$

$$= 2{,}636\,66 \cdot 10^{-4}\ \mathrm{J \cdot m^{-1}}$$

9.20



$$d\phi = d(\vec{B}\cdot\vec{S})$$

pre $\vec{B} \perp \vec{S}$

platí $d\phi = B\,dS$, čiže v každom mieste plôšky je indukcia konštantná!

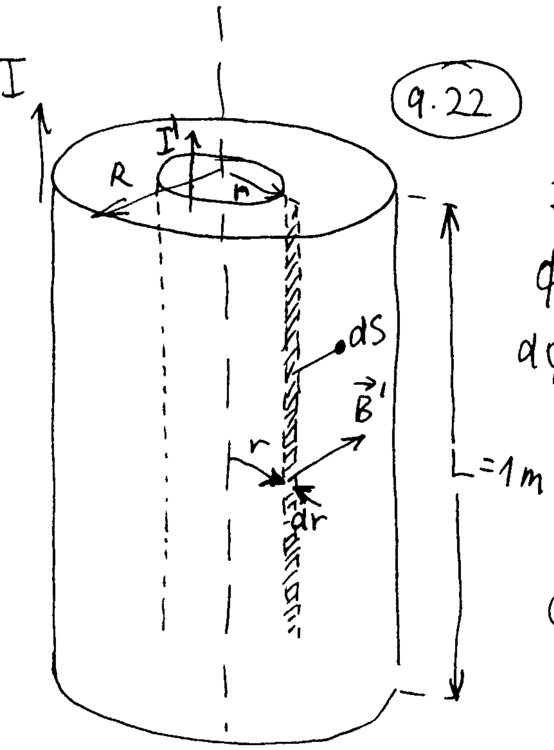
$$dS = b \cdot dx$$

$$d\phi = \frac{A}{x}\, b\, dx$$

$$\phi = A\cdot b \int_{c}^{a+c} \frac{dx}{x} = A\cdot b\ \mathrm{Ln}\left|\frac{c+a}{c}\right| = 10^{-4}\cdot 10\cdot 10^{-2}\ \mathrm{Ln}\left|\frac{18}{10}\right| =$$

$$= 10^{-5}\ \mathrm{Ln}\,|1{,}8| = \underline{0{,}587\,787 \cdot 10^{-5}\ \mathrm{Wb}}$$

9.22



$$I' = jS = j\pi r^2$$

$$\phi = |\vec{B}|\cdot|\vec{S}|\cos\alpha = B\cdot S$$

$$d\phi = B\cdot dS = \frac{\mu_0 I'}{2\pi r} L\,dr = \frac{\mu_0 j \pi r^2}{2\pi r} L\,dr =$$

$$= \frac{\mu_0 j L}{2\pi}\, r\,dr$$

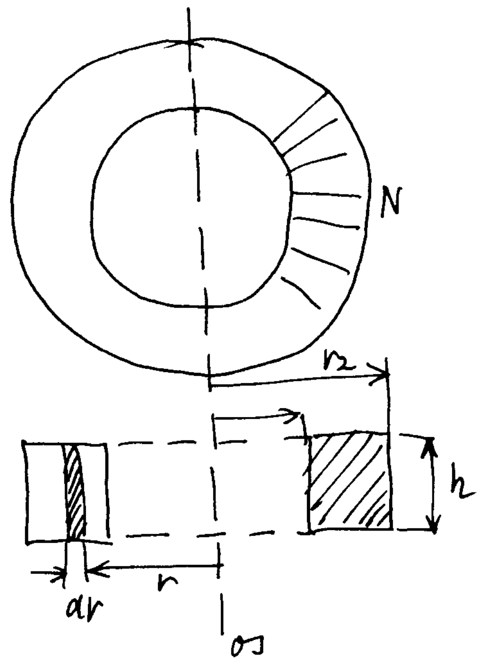
$$\phi = \frac{\mu_0 j L}{2\pi}\int_0^R r\,dr = \frac{\mu_0 j L}{2}\cdot\frac{R^2}{2} = \frac{\mu_0 R^2}{4}\, jL =$$

$$= \frac{\mu_0 \pi R^2}{4\pi}\, jL = \frac{\mu_0 I}{4\pi}\, L$$

$$\Downarrow$$

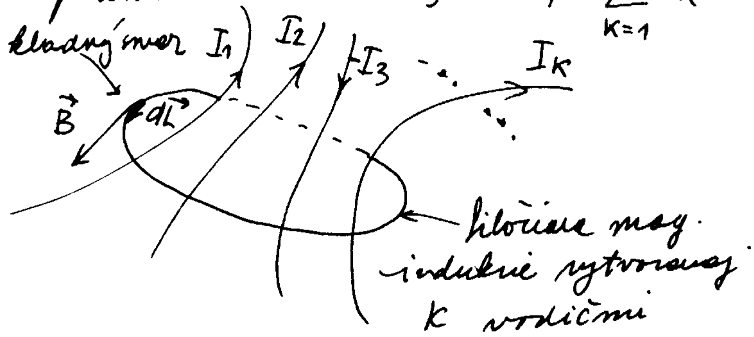
$$\frac{\phi}{L} = \frac{\mu_0 I}{4\pi}$$

9.23



Na riešenie použijeme zákon celkového prúdu. Pre N vodičov platí:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \sum_{k=1}^{N} I_k$$



V mieste vo vzdialenosti r od osi platí:

$$B \oint dl = \mu_0 N I$$

$$B \cdot 2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

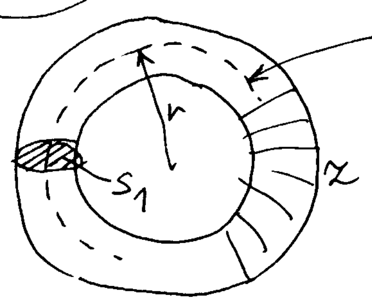
$$dS = h.dr \,, \quad \frac{r_2}{r_1} = \alpha$$

$$d\phi = B.dS = \frac{\mu_0 N I}{2\pi} h \frac{dr}{r}$$

$$\phi = \frac{\mu_0 N I}{2\pi} h \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 N I}{2\pi} h \, \ln\left|\frac{r_2}{r_1}\right| =$$

$$= \frac{4\pi\cdot 10^{-7}\cdot 10^{3}\cdot 1{,}7\cdot 5\cdot 10^{-2}}{2\pi} \ln 1{,}6 = \underline{7{,}990\,062\cdot 10^{-6}\,Wb}$$

9.24



$$B_{str} = \frac{\mu_r \mu_0 z I}{2\pi r_{str}} = \frac{200\cdot 4\pi\cdot 10^{-7}\cdot 1500\cdot 1}{2\pi\cdot 10\cdot 10^{-2}} =$$

$$= 5{,}999\,999\cdot 10^{-1}\,T \approx \underline{0{,}6\,T}$$

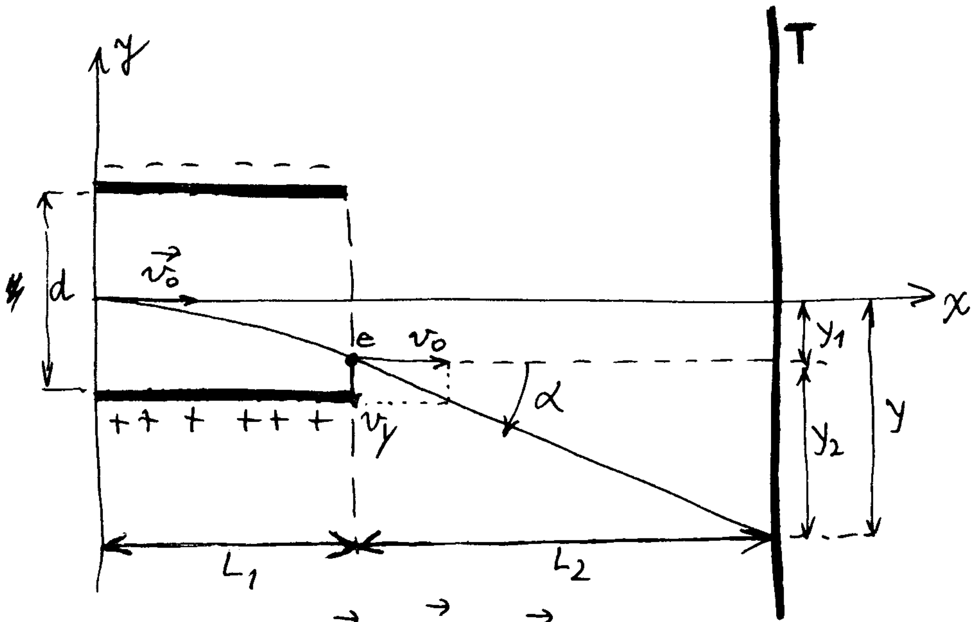
$$\phi = B_{str}\cdot S_1$$

$$\phi_{celk} = z\cdot\phi = z\cdot B_{str}\cdot S_1 = \frac{\mu_r \mu_0 z^2 I\cdot S_1}{2\pi r_{str}} =$$

$$= \frac{200\cdot 4\pi\cdot 10^{-7}\cdot 1500^2\cdot 1\cdot 4\cdot 10^{-4}}{2\pi\cdot 10\cdot 10^{-2}} =$$

$$= 3{,}599\,999\cdot 10^{-1} \approx \underline{0{,}36\,Wb}$$

(9.27)



$$\vec{F} = q\vec{E} = m\vec{a}$$

$$\vec{a} = \frac{q\vec{E}}{m}, \quad t = \frac{L_1}{v_0}, \quad |\vec{E}| = \frac{U}{d}, \quad q = e$$

$$tg\,\alpha = \frac{v_y}{v_0} = \frac{\frac{q}{m}E \cdot t}{v_0} = \frac{qE \cdot L_1}{m v_0^2}$$

$$y_1 = \frac{q}{m}E\frac{t^2}{2} = \frac{qE}{2m} \cdot \frac{L_1^2}{v_0^2}$$

$$y_2 = L_2\, tg\,\alpha = \frac{qEL_1L_2}{m v_0^2}$$

$$y = y_1 + y_2 = \frac{qEL_1^2}{2m v_0^2} + \frac{qEL_1L_2}{m v_0^2} =$$

$$= \frac{L_1 qE}{m v_0^2}\left(\frac{L_1}{2} + L_2\right) = \frac{L_1 q U}{m v_0^2 d}\left(\frac{L_1}{2} + L_2\right) =$$

$$= \frac{3 \cdot 10^{-2} \cdot 1{,}602 \cdot 10^{-19} \cdot 100}{9{,}1 \cdot 10^{-31} (10^7)^2 \cdot 10^{-2}}\left(\frac{3 \cdot 10^{-2}}{2} + 30 \cdot 10^{-2}\right) =$$

$$= 1{,}663\,615 \cdot 10^{-1}\,m \approx 16{,}6\,cm$$

$$|\vec{a}| = \frac{qE}{m} = \frac{dv_y}{dt}$$

$$dv_y = \frac{q}{m}E\,dt$$

$$v_y = \frac{q}{m}E\int dt + C_1$$

$$v_y = \frac{q}{m}Et + C_1$$

Pre $t = \emptyset$, $v_y = \emptyset$

$$\emptyset = \frac{q}{m}E \cdot \emptyset + C_1 \Rightarrow C_1 = \emptyset$$

- $v_y = \frac{q}{m}E \cdot t = \frac{dy}{dt}$

$$dy = \frac{q}{m}Et\,dt$$

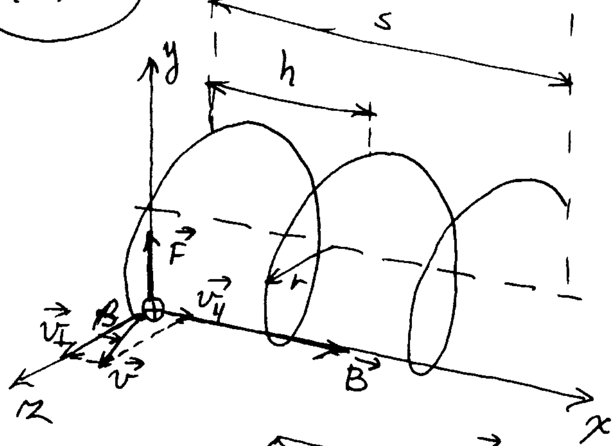
$$y = \frac{q}{m}E\int t\,dt + C_2$$

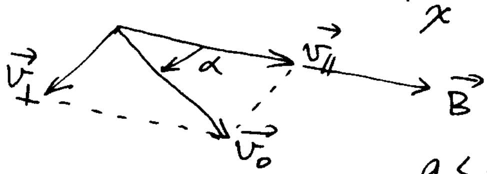
- $y = \frac{q}{m}E\frac{t^2}{2} + C_2$

Pre $t = \emptyset$, $y = \emptyset$

$$C_2 = \emptyset$$

9.29





$q < 0$, $B = konšt.$

$$h = v_\parallel T \quad , \quad T = \frac{2\pi r}{v_\perp}$$

$$|\vec{v_\perp}| = v_0 \cdot \sin\alpha$$

$$|\vec{v_\parallel}| = v_0 \cdot \cos\alpha$$

$$\left.\begin{aligned} F_L &= |q|\, v B \\ F_d &= \frac{m v^2}{r} \end{aligned}\right\} \; r = \left|\frac{m}{q}\right| \frac{v}{B}$$

Rýchlosť $v_\parallel$ sa v mag. poli nemení. Častica vykonáva súčasne dva pohyby: rovnomerný po kružnici s polomerom r rýchlosťou $v_\perp$ a postupný v smere indukcie B rýchlosťou $v_\parallel$.

$$r = \left|\frac{m}{q}\right| \frac{v_\perp}{B} = \left|\frac{m}{q}\right| \frac{v_0 \sin\alpha}{B} = \frac{9{,}1 \cdot 10^{-31}}{1{,}602 \cdot 10^{-19}} \cdot \frac{10^4 \cdot \sin 30^\circ}{0{,}01} =$$

$$= \underline{\underline{2{,}840\,199 \cdot 10^{-6}\,m}}$$

$$h = v_\parallel \frac{2\pi r}{v_\perp} = v_0 \cos\alpha \frac{2\pi r}{v_0 \sin\alpha} = 2\pi r \operatorname{ctg}\alpha = 2\pi \cdot 2{,}84 \cdot 10^{-6} \cdot 1{,}732\,051 =$$

$$= \underline{\underline{3{,}090\,931 \cdot 10^{-5}\,m}}$$

$$t = nT = \frac{s}{h} T = \frac{s}{h} \cdot \frac{2\pi}{B} \left|\frac{m}{q}\right| = \frac{1}{3{,}090\,931 \cdot 10^{-5}} \cdot \frac{2\pi}{0{,}01} \cdot \frac{9{,}1 \cdot 10^{-31}}{1{,}602 \cdot 10^{-19}} =$$

$$= \underline{\underline{1{,}154\,700 \cdot 10^{-4}\,s}}$$