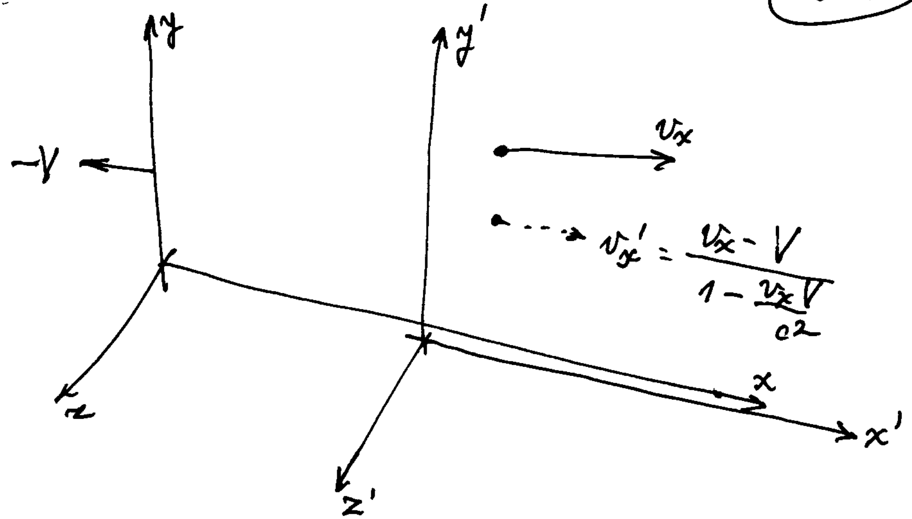
5.11



$$v_x' = \frac{v_x - V}{1 - \frac{v_x V}{c^2}}$$

$$x' = \gamma x - \gamma v t$$
$$t' = \gamma t - \frac{\gamma}{c^2} v x$$

$$dx' = \gamma dx - \gamma v dt$$
$$dt' = \gamma dt - \frac{\gamma}{c^2} v dx$$

$$v_x' = \frac{dx'}{dt'} = \frac{\gamma dx - \gamma v dt}{\gamma dt - \frac{\gamma}{c^2} v dx} = \frac{\frac{dx}{dt} - v}{1 - \frac{\frac{dx}{dt} \cdot v}{c^2}} =$$

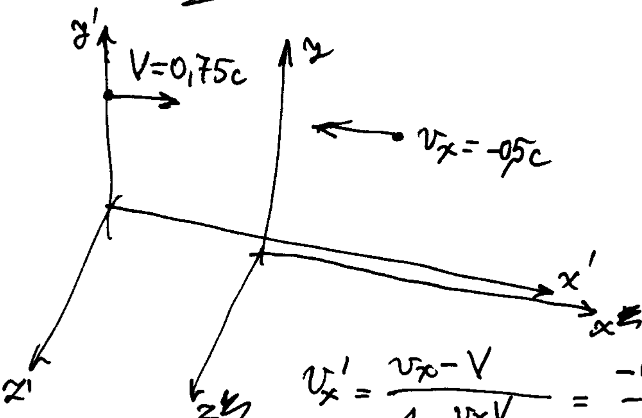
$$= \frac{v_x - v}{1 - \frac{v_x v}{c^2}} = \left| \begin{array}{l} \beta = \frac{V}{c} \\ v = V \end{array} \right. = \frac{v_x - V}{1 - \frac{v_x V}{c^2}}$$

$$x = \gamma x' + \gamma \beta c t'$$
$$t = \gamma t' + \gamma \beta x'/c$$

$$dx = \gamma\, dx' + \gamma \beta c\, dt'$$
$$dt = \gamma\, dt' + \gamma \beta\, dx'/c$$

$$v_x = \frac{dx}{dt} = \frac{\gamma dx' + \gamma \beta c\, dt'}{\gamma dt' + \gamma \beta\, dx'/c} = \frac{v_x' + \beta c}{1 + \frac{\beta v_x'}{c}} =$$

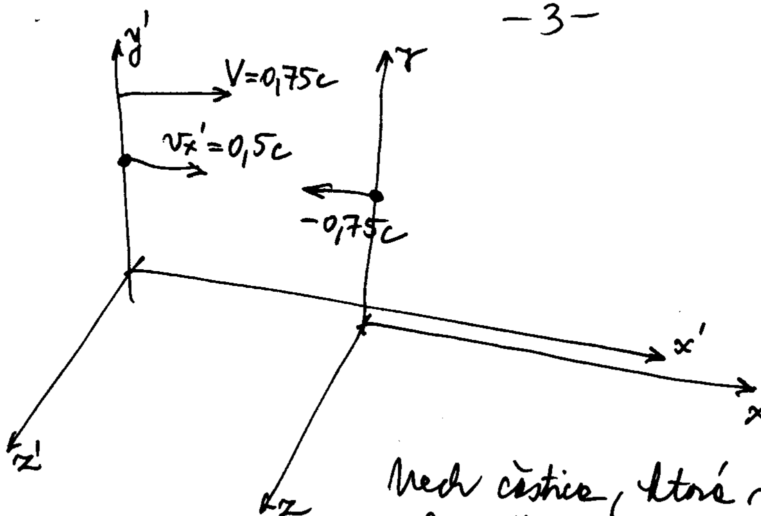
$$= \frac{v_x' + V}{1 + \frac{v_x' V}{c^2}}$$



$$v_x' = \frac{v_x - V}{1 - \frac{v_x V}{c^2}} = \frac{-0{,}5c - 0{,}75c}{1 - \frac{(-0{,}5) \times 0{,}75 c^2}{c^2}} =$$

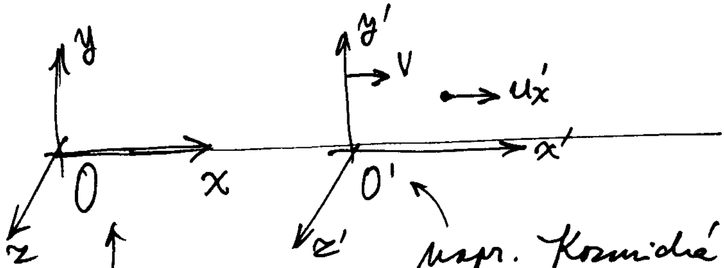
$$= \frac{-1{,}25c}{1 + 0{,}375} = \frac{-1{,}25}{1{,}375} c = -0{,}\overline{09}09c \approx \underline{\underline{-0{,}91c}}$$

Znamienko mínus označuje (znamená), že rýchlosť má smer doľava, t.j. záporný smer osi x'.



Nech častica, ktorá má rýchlosť −0,75c je nepohyblivá, potom rýchlosť systému S' je rovná V=+0,75c.
Častica, ktorá má rýchlosť $v_x' = 0{,}5c$ v systéme S' má v systéme S rýchlosť

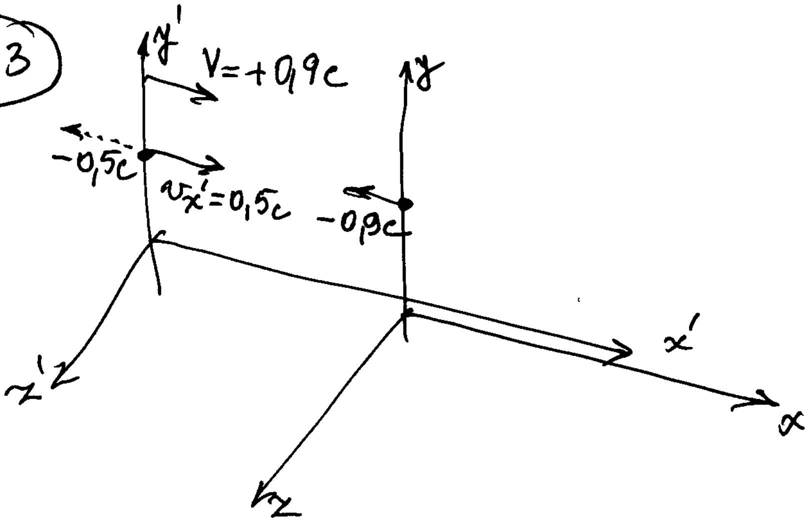
$$v_x = \frac{v_x' + V}{1 + \frac{v_x' V}{c^2}} = \frac{0{,}5c + 0{,}75c}{1 + 0{,}375} = \frac{1{,}25}{1{,}375}c \approx 0{,}91c$$



napr. Zem

napr. Kozmická loď letiaca rýchlosťou V, z ktorej je vystrelená raketa rýchlosťou u_x'.

5.13



$$v_x = \frac{v_x' + V}{1 + \frac{v_x' V}{c^2}} = \frac{0,5c + 0,9c}{1 + 0,45} = \frac{1,4}{1,45} = 0,9655c \approx \underline{0,97c}$$

$$v_x = \frac{-0,5c + 0,9c}{1 - 0,45} = \frac{0,4c}{0,55} = \underline{0,\overline{72}c} \approx \underline{0,73c}$$

5.17

$E_k = 35\,MeV$

$E_0 = 140\,MeV$

$$E = E_k + E_0$$

$$\frac{m_0 c^2}{\sqrt{1-\beta^2}} = E_k + E_0$$

$$\frac{E_0}{\sqrt{1-\beta^2}} = E_k + E_0$$

$$\frac{1}{\sqrt{1-\beta^2}} = \frac{E_k + E_0}{E_0} = \frac{\Delta t}{\Delta t_0}$$

$$\frac{35+140}{140} = \frac{\Delta t}{\Delta t_0}$$

$$\frac{\Delta t}{\Delta t_0} = \frac{175}{140} = \underline{\underline{1,25}}$$

5.20

$$E = E_K + E_0 \Rightarrow E_K = E - E_0$$

$$E_K = \frac{m_0 c^2}{\sqrt{1-\beta^2}} - m_0 c^2 = 7 m_0 c^2$$

$$\Downarrow$$

$$\frac{1}{\sqrt{1-\beta^2}} - 1 = 7$$

$$\frac{1}{\sqrt{1-\beta^2}} = 8$$

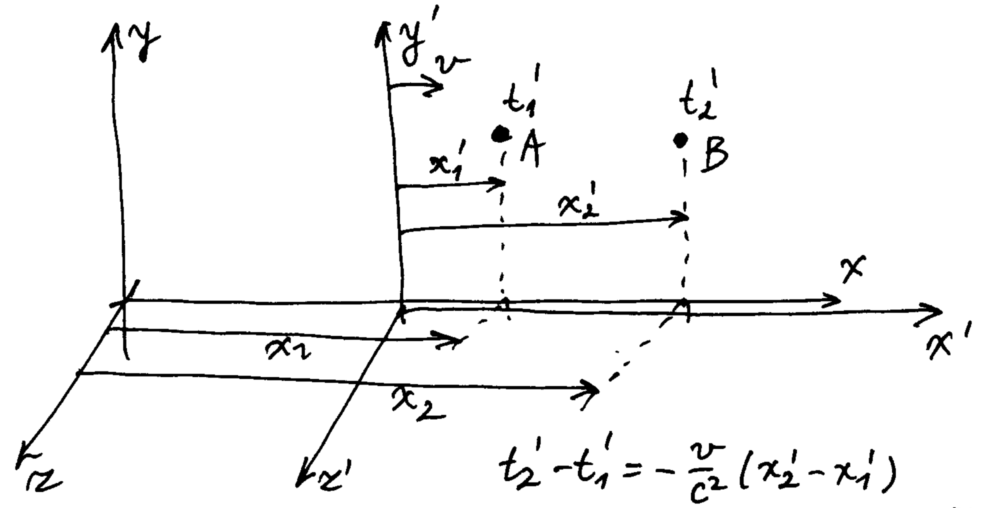
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\beta^2}} \Rightarrow \Delta t_0 = \Delta t \sqrt{1-\beta^2}$$

$$\Delta t_0 = \frac{\Delta t}{8} = \frac{1{,}76 \cdot 10^{-5} s}{8}$$

$$\underline{\underline{\Delta t_0 = 2{,}2 \cdot 10^{-6} s}}$$

__Udalosť__ (bodová) je definovaná usporiadanou štvoricou súradníc $[x, y, z, t]$.
Dve udalosti $U_1[x_1, y_1, z_1, t_1]$ a $U_2[x_2, y_2, z_2, t_2]$, pre ktoré platí: $t_1 = t_2$ sú __súčasné__,
$[x_1, y_1, z_1] = [x_2, y_2, z_2]$ sú __súmiestne__, ak nastali na tom istom mieste; v tom istom bode.

__Relatívnosť súčasnosti:__ Udalosti U_1 a U_2, ktoré sú súčasné vzhľadom na jednu inerciálnu sústavu S_1, vo všeobecnosti nie sú súčasné vzhľadom na inú inerciálnu sústavu S_2.
__Súčasnosť__ dvoch udalostí je __relatívny pojem__.
[Súčasnosť nie je absolútny pojem].



$$t_2' - t_1' = -\frac{v}{c^2}(x_2' - x_1')$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\boxed{\begin{array}{l} x_2' - x_1' = \gamma (x_2 - x_1) \\ v \text{ prípade, že } t_1 = t_2 \end{array}}$$

$$\Delta x' = x_2' - x_1' = \gamma\left[(x_2 - x_1) - v(t_2 - t_1)\right] = \gamma\left[L_0 - v(t_2 - t_1)\right] = \gamma\left[\Delta x - v\Delta t\right]$$

$$\Delta t' = t_2' - t_1' = \gamma\left[\left(t_2 - \frac{v x_2}{c^2}\right) - \left(t_1 - \frac{v x_1}{c^2}\right)\right] = \gamma\left[\underbrace{(t_2 - t_1)}_{\Delta t} - \frac{v}{c^2}\underbrace{(x_2 - x_1)}_{\Delta x}\right] =$$

$$= \gamma\left[\Delta t - \frac{v}{c^2}\Delta x\right]$$

$$\Delta x = \frac{\Delta x'}{\gamma} + v\Delta t$$

$$\Delta t' = \gamma\left[\Delta t + \frac{v}{c^2}\left(\frac{\Delta x'}{\gamma} + v\Delta t\right)\right] = \gamma\Delta t - \frac{v\Delta x'}{c^2} - \gamma\frac{v^2}{c^2}\Delta t = \gamma\Delta t\left(1 - \frac{v^2}{c^2}\right) - \frac{v\Delta x'}{c^2}$$

$$\Delta t' + \frac{v}{c^2}\Delta x' = \gamma\,\Delta t\left(1 - \frac{v^2}{c^2}\right)$$

$$\Delta t = \frac{\Delta t' + \frac{v}{c^2}\Delta x'}{\gamma\left(1-\frac{v^2}{c^2}\right)} = \frac{\Delta t' + \frac{v}{c^2}\Delta x'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$v = \frac{3}{5} . d . s^{-1}$, $\Delta t' = 12 s$, $\Delta x' = 4 . d$, $d = 3 . 10^8 m$

$$\Delta t = \frac{12 + \frac{\overbrace{\frac{3}{5} d . s^{-1}}^{\sim c}}{c^2} . 4d}{\sqrt{1-\frac{(\frac{3}{5}c)^2}{c^2}}} = \frac{12 + \frac{3}{5} . \frac{4 . 3 . 10^8}{3 . 10^8}}{\sqrt{1-\frac{9}{25}}} =$$

$$= \frac{\frac{5}{5}12 + \frac{12}{5}}{\sqrt{\frac{16}{25}}} = \frac{\frac{5 . 12 + 12}{5}}{\frac{4}{5}} = \frac{5 . (5 . 12 + 12)}{5 . 4} =$$

$$= \frac{5 . 12 + 12}{4} = \frac{72}{4} = \underline{\underline{18 s}}$$

5.9

$\Delta t = 1 s$, $\Delta x = 6 . 10^5 km$, $v = ?$

$\Delta t' = t_2' - t_1' = \emptyset$ - podmienka súčasnosti dopadov z pohľadu kozmonauta

$$t_2 - t_1 = \frac{v}{c^2}(x_2 - x_1)$$

$$v = \frac{t_2 - t_1}{x_2 - x_1} c^2 = \frac{\Delta t}{\Delta x} c^2 = \frac{1 s}{6 . 10^8} . 9 . 10^{16} = \underline{\underline{1,5 \times 10^8 m s^{-1}}}$$

5.12

u_y' ; $u_x' = \emptyset$

$$u_y = \frac{u_y' \sqrt{1-\beta^2}}{\underbrace{1 + \frac{v u_x'}{c^2}}_{1}} \quad ; \quad u_x = v$$

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$

$\cancel{u = u_y}$ $u = u_y^2 + u_x^2 =$

- Keď udalosti A a B nastanú v jednom bode $x_1 = x_2$, v tom istom čase $t_1 = t_2$, <u>budú súčasnými</u> aj v ľubovoľnej inej súr. sústave!

- Keď $t_1 = t_2$ a $x_1 \neq x_2$, potom <u>nebudú súčasnými</u> v ľubovoľných iných inerciálnych súr. sústavách.

$$t_2' - t_1' = \frac{(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)}{\sqrt{1-\beta^2}} = \gamma (t_2 - t_1) - \frac{v}{c^2}\gamma (x_2 - x_1)$$

$$\boxed{t_1 = t_2} \Longrightarrow t_2' - t_1' = -\frac{v}{c^2}\gamma (x_2 - x_1) = \frac{-\frac{v}{c^2}(x_2 - x_1)}{\sqrt{1-\beta^2}} \Rightarrow$$

$$t_1' - t_2' = \frac{x_2 - x_1}{c\sqrt{\frac{c^2}{v^2} - 1}} = \frac{x_2 - x_1}{c\sqrt{\beta^2 - 1}} .$$

<u>Keď $x_2 > x_1$, potom $t_2' < t_1'$.</u>

5.15

$$\rho = \frac{m}{V} = \frac{\frac{m_0}{\sqrt{1-\beta^2}}}{\Delta x \cdot \Delta y \cdot \Delta z} = \frac{\frac{m_0}{\sqrt{1-\beta^2}}}{\Delta x'\sqrt{1-\beta^2} \cdot \Delta y \cdot \Delta z}$$

$$\rho_0 = \frac{m_0}{V_0} = \frac{m_0}{\Delta x' \cdot \Delta y' \cdot \Delta z'}$$

$$\Delta y = \Delta y'$$
$$\Delta z = \Delta z'$$
$$\Delta x = \Delta x'\sqrt{1-\beta^2}$$

$$\underbrace{\frac{\rho - \rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = 0{,}1}_{\Downarrow} \qquad (10\%)$$

$$\frac{\frac{\frac{m_0}{\sqrt{1-\beta^2}}}{\Delta x'\sqrt{1-\beta^2} \cdot \Delta y \Delta z}}{\frac{m_0}{\Delta x' \cdot \Delta y' \cdot \Delta z'}} - 1 = 0{,}1$$

$$\frac{m_0 \Delta x' \Delta y' \Delta z'}{m_0 \Delta x' \Delta y \Delta z (1-\beta^2)} - 1 = 0{,}1$$

$$\frac{1}{1-\left(\frac{v}{c}\right)^2} - 1 = 0{,}1$$

$$\frac{1}{1-\left(\frac{v}{c}\right)^2} = 0{,}1 + 1$$

$$\frac{1}{\frac{1}{10}+1} = 1 - \left(\frac{v}{c}\right)^2$$

$$1 - \frac{10}{11} = \left(\frac{v}{c}\right)^2$$

$$\frac{1}{11} = \left(\frac{v}{c}\right)^2$$

$$v = \frac{c}{\sqrt{11}} \approx 0{,}3\,c$$

5.2

$v = 0{,}98c$

$\Delta t_0 = 1\,min$

$\Delta t = ?$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\beta^2}} = \frac{1\,min}{\sqrt{1-0{,}98^2}} = 5\,min$$

5.4

$\Delta l = 100\,m$

$\Delta L_0 = 99\,m$

$$\Delta L_0 = \Delta L\sqrt{1-\beta^2}$$

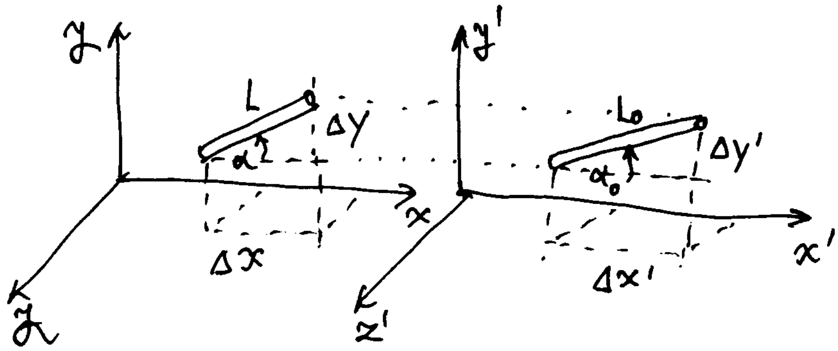
$$\left(\frac{\Delta L_0}{\Delta L}\right)^2 = 1-\beta^2$$

$$\beta^2 = 1 - \left(\frac{\Delta L_0}{\Delta L}\right)^2$$

$$v = c\sqrt{1-\left(\frac{\Delta L_0}{\Delta L}\right)^2} = 3\cdot 10^8\sqrt{1-\left(\frac{99}{100}\right)^2} =$$

$$= 4{,}232 \cdot 10^7\,m\,s^{-1}$$

5.06



$$\Delta x = \Delta x' \sqrt{1-\beta^2} \qquad L = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\Delta y = \Delta y' \Rightarrow L_0 \sin\alpha_0 = L \sin\alpha$$

$$L_0 = \sqrt{(\Delta x')^2 + (\Delta y')^2} = \sqrt{\left(\frac{\Delta x}{\sqrt{1-\beta^2}}\right)^2 + (\Delta y')^2} =$$

$$= \sqrt{\frac{\Delta x^2 + \Delta y'^2 - \beta^2 \Delta y'^2}{1-\beta^2}} = \left| \begin{array}{l} \Delta y = \Delta y' \\ L^2 = \Delta x^2 + \Delta y^2 \end{array} \right| = \sqrt{\frac{L^2 - \beta^2 \Delta y^2}{1-\beta^2}} =$$

$$= \frac{L\sqrt{1-\beta^2 \sin^2\alpha}}{\sqrt{1-\beta^2}} = \left| \begin{array}{l} v = \frac{c}{2} \\ \alpha = 45^\circ \\ \beta = \frac{v}{c} = \frac{\frac{c}{2}}{c} = \frac{1}{2} \\ L = 1m \end{array} \right.$$

$$= \frac{1\sqrt{1-\left(\frac{1}{2}\right)^2 \sin^2 45^\circ}}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{1\sqrt{1-\frac{1}{8}}}{\sqrt{1-\frac{1}{4}}} = \frac{\sqrt{\frac{7}{8}}}{\sqrt{\frac{3}{4}}} =$$

$$= \sqrt{\frac{28}{24}} = 1{,}08\ m$$