4.1

$$F_1 = k x_1 \Rightarrow k = \frac{F_1}{x_1}$$

$$\left.\begin{aligned} F &= m\frac{d^2x}{dt^2} \\ x &= x_0 \sin(\omega t + \varphi) \end{aligned}\right\} \frac{d^2x}{dt^2} = -\omega^2 x$$

$$\omega^2 m = k$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F_1}{m x_1}}$$

4.2

$$x = A\cos\omega t$$

$$v = \frac{dx}{dt} = -\underbrace{A\omega}_{v_{max}} \sin\omega t$$

$$\frac{v_{max}}{2} = -v_{max}\sin\omega t$$

$$\arcsin\left(-\frac{1}{2}\right) = \omega t \quad ; \quad \omega = \frac{2\pi}{T}$$

$$t = \frac{\arcsin\left(-\frac{1}{2}\right)}{\frac{2\pi}{T}} = \frac{\frac{\pi}{6}}{\frac{2\pi}{T}} = \frac{T}{12}$$

4.9

$$x = x_0 \sin \omega t$$

$$F = kx \Rightarrow x = \frac{F}{k}$$

$$E = E_p + E_K = \frac{1}{2} k x_0^2 \Rightarrow k = \frac{2E}{x_0^2}$$

$$x = \frac{F}{\frac{2E}{x_0^2}} = \underline{\underline{\frac{x_0^2 F}{2E}}}$$

4.10

$$\frac{E_p}{E_k} = \frac{\frac{1}{2}kx^2}{\frac{1}{2}mv^2} = \qquad x = A\cos\omega t, \quad v = -A\omega \sin\omega t$$

$$m\omega^2 = k$$

$$= \frac{\frac{1}{2} k A^2 \cos^2 \omega t}{\frac{1}{2} m A^2 \omega^2 \sin^2 \omega t} = \frac{\cos^2 \omega t}{\sin^2 \omega t} = ctg^2(\omega t) = ctg^2\left(\frac{2\pi}{T} \cdot \frac{T}{8}\right) =$$

$$= ctg^2\left(\frac{\pi}{4}\right) = \underline{1}$$

4.11

$$\boxed{W = E_p} \qquad k = \frac{mg}{s_0}$$

Práca potrebná na stlačenie pružiny $\longrightarrow$ $\frac{1}{2} k s_1^2 = m g h_{max}$ $\longleftarrow$ Max. polohová energia guľôčky, ktorú získa po vystrelení pružinou.

$$\frac{1}{2} \frac{mg}{s_0} s_1^2 = m g h_{max}$$

$$\underline{\underline{h_{max} = \frac{s_1^2}{2 s_0}}}$$

4.15

koef. tlmenia

$$x = A e^{-\delta t} \cos(\omega t + \varphi)$$

útlm $\lambda = \dfrac{x(t)}{x(t+T)} = e^{\delta T}$

Logaritmický dekrement útlmu $\ln \lambda = \delta T = 0{,}02$

$\varphi = 0$

$$\frac{x(t)}{x(t+100T)} = \frac{A e^{-\delta t} \cos \omega t}{A e^{-\delta(t+100T)} \cos[\omega(t+100T)]}$$

pre $t=0$ $\quad \dfrac{x_0}{x_{100}} = \dfrac{1}{e^{-100\delta T}} = e^{100\delta T} = \underline{\underline{e^2}}$

4.17

$u(0,t) = A \cos \omega t \quad ; \quad \lambda = v T$

$u(r,t) = A \cos 2\pi f\left(t - \frac{r}{v}\right) = A \cos 2\pi \left(\frac{t}{T} - \frac{r}{\lambda}\right)$

$\dfrac{A}{2} = A \cos 2\pi \left(\frac{t}{T} - \frac{r}{\lambda}\right) \quad ; \quad t = \dfrac{T}{3} \quad , \quad r = 4\,\text{cm}$

$$\frac{\arccos\left(\frac{1}{2}\right)}{2\pi} = \frac{t}{T} - \frac{r}{\lambda}$$

$$\frac{r}{\lambda} = \frac{t}{T} - \frac{\arccos\left(\frac{1}{2}\right)}{2\pi}$$

$$\lambda = \frac{r}{\frac{t}{T} - \frac{\arccos\left(\frac{1}{2}\right)}{2\pi}} = \frac{4\,\text{cm}}{\frac{1}{3} - \frac{1}{6}} =$$

$$= \frac{4\,\text{cm}}{\frac{1}{6}} = 24\,\text{cm}$$

4.21

$$u(r_1,t) = A\cos 2\pi f\left(t-\frac{r_1}{v}\right) = A\cos 2\pi\left(\frac{t}{T}-\frac{r_1}{vT}\right)$$

$$u(r_2,t) = A\cos 2\pi\left(\frac{t}{T}-\frac{r_2}{vT}\right)$$

$$\Delta\varphi = \frac{2\pi}{T}\left[\left(t-\frac{r_1}{v}\right)-\left(t-\frac{r_2}{v}\right)\right] = \frac{2\pi}{T}\left[\frac{r_2}{v}-\frac{r_1}{v}\right] =$$

$$= \frac{2\pi}{\lambda}(r_2-r_1) \overset{\text{alebo}}{=} \frac{2\pi}{vT}(r_2-r_1)$$

4.19

$$u(r,t) = 60\sin(1800t-5{,}3r)\ [\mu m]$$

$$u(r,t) = A\sin\left(\frac{2\pi}{T}t-\frac{2\pi}{vT}r\right)$$

$$\frac{2\pi}{vT} = \frac{\omega}{v} = 5{,}3$$

$$\boxed{v = \frac{\omega}{5{,}3}}$$

$$A = 60\,\mu m$$

$$\frac{2\pi}{T} = 1800\ \text{rad}\ s^{-1}$$

$$\frac{2\pi}{\lambda} = \frac{2\pi}{vT} = 5{,}3\ \text{rad}\ m^{-1} \Rightarrow \lambda = \ldots$$

$$\frac{A}{\lambda} = \ldots$$

$$v_{max} = A\omega = A\frac{2\pi}{T} = \ldots$$

$$\frac{v_{max}}{v} = \frac{A\frac{2\pi}{T}}{\frac{\omega}{5{,}3}} = \frac{A\omega}{\frac{\omega}{5{,}3}} = \underline{5{,}3\cdot A}$$