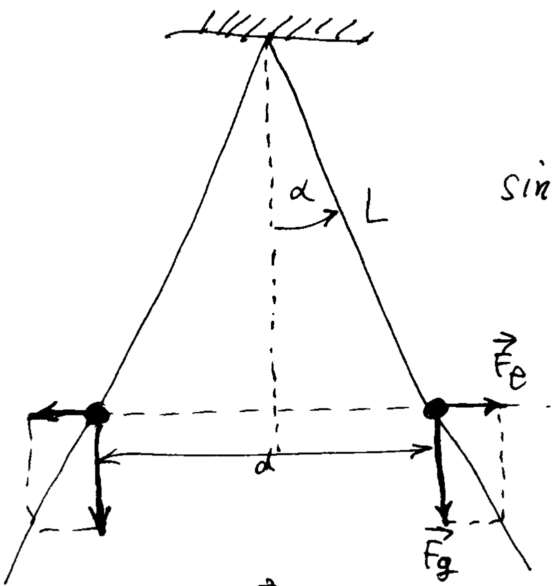
## 7.15




$$\sin\alpha = \frac{\left(\frac{d}{2}\right)}{L} = \frac{d}{2L}$$

$$\frac{L\cdot\cos\alpha}{\left(\frac{d}{2}\right)} = \frac{|\vec{F_g}|}{|\vec{F_e}|} = \frac{mg}{\frac{1}{4\pi\varepsilon}\frac{q^2}{d^2}} = \frac{mg}{q^2}4\pi\varepsilon d^2$$

$$2L\cos\alpha = \frac{mg}{q^2}4\pi\varepsilon d^3$$

$$\cos^2\alpha = 1-\sin^2\alpha$$
$$\cos^2\alpha = 1-\left(\frac{d}{2L}\right)^2$$

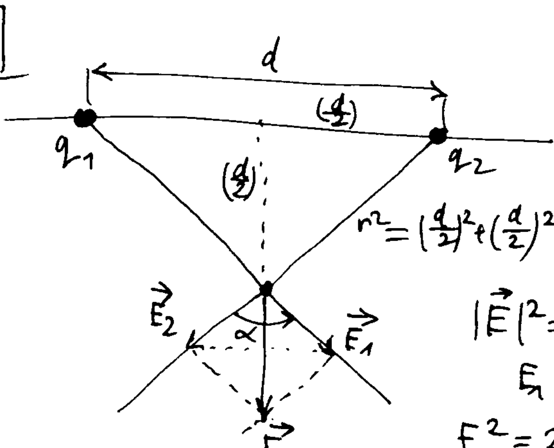
$$q^2 = \frac{mg\,4\pi\varepsilon d^3}{2L\cos\alpha} =$$

$$= \frac{mg\,4\pi\varepsilon d^3}{2L\sqrt{1-\left(\frac{d}{2L}\right)^2}} = \frac{5\cdot10^{-4}\cdot9{,}806\cdot(4\cdot10^{-2})^3\cdot4\cdot\pi\cdot8{,}854\cdot10^{-12}}{2\cdot1\sqrt{1-\left(\frac{4\cdot10^{-2}}{2\cdot1}\right)^2}} =$$

$$= 1{,}746016\cdot10^{-17}\,C^2$$

$$q = 4{,}178535\cdot10^{-9}\,C$$

## 7.18




$$r^2 = \left(\frac{d}{2}\right)^2+\left(\frac{d}{2}\right)^2 = 2\left(\frac{d}{2}\right)^2$$

$$|\vec{E}|^2 = E_1^2+E_2^2+2E_1E_2\cos\alpha$$

$$E_1 = E_2$$

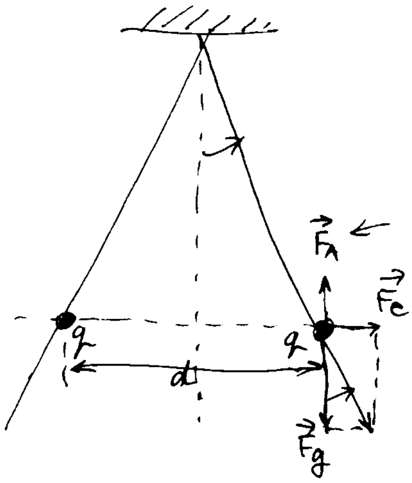
$$E^2 = 2E_1^2$$

$$E = \sqrt{2}E_1 = \frac{\sqrt{2}\,q}{4\pi\varepsilon\cdot r^2} = \frac{\sqrt{2}\,q}{4\pi\varepsilon\,2\left(\frac{d}{2}\right)^2} =$$

$$= \frac{\sqrt{2}\,q}{2\pi\varepsilon d^2} = \frac{\sqrt{2}\cdot10^{-6}}{2\pi\cdot8{,}854\cdot10^{-12}\cdot1^2} =$$

$$= 2{,}542117\cdot10^{4}\,V\cdot m^{-1}$$

7.16




Pri ponorení do kvapaliny na guľôčky pôsobí Archimedova sila.

Vo vzduchu: $|\vec{F}_g| = mg$ , $|\vec{F}_e| = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

V kvapaline: $|\vec{F}_g'| = mg - F_A$ , $|\vec{F}_e'| = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q^2}{d^2}$

Archimedova sila

$$\frac{\left(\frac{d}{2}\right)}{L\sin\alpha} = \frac{|\vec{F}_e|}{|\vec{F}_g|} = \frac{|\vec{F}_e'|}{|\vec{F}_g'|}$$

$$\frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{d^2}}{mg} = \frac{\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q^2}{d^2}}{mg - F_A}$$

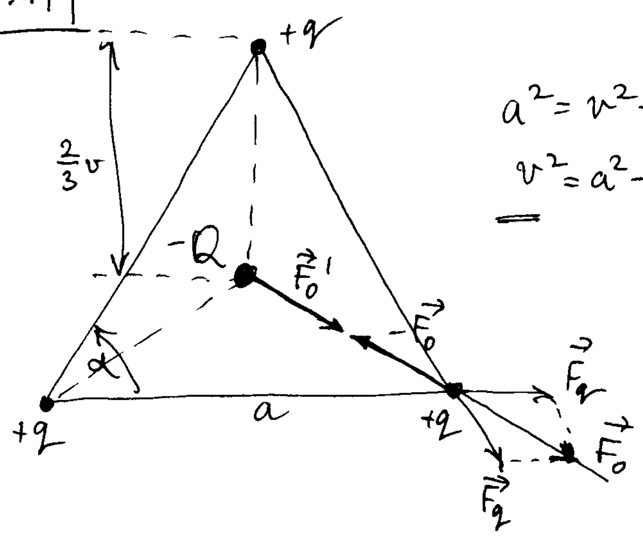
$$\frac{1}{mg} = \frac{\frac{1}{\varepsilon_r}}{mg - F_A} \Rightarrow F_A = mg - \frac{mg}{\varepsilon_r}$$

$$F_A = mg\left(1 - \frac{1}{\varepsilon_r}\right)$$

$$\frac{4}{3}\pi R^3 \rho g = mg\left(1 - \frac{1}{\varepsilon_r}\right)$$

$$\rho = \frac{m}{\frac{4}{3}\pi R^3}\left(1 - \frac{1}{\varepsilon_r}\right) = \frac{1\cdot 10^{-3}}{\frac{4}{3}\pi\,(3\cdot 10^{-3})^3}\left(1 - \frac{1}{1{,}1}\right) = 0{,}8038128\cdot 10^{3}\,\frac{kg}{m^3}$$

$\boxed{7.17}$




$$a^2 = v^2 + \left(\frac{a}{2}\right)^2$$

$$v^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$$

$$k = \frac{1}{4\pi\varepsilon}$$

$$|\vec{F}_0| = 2F_q \cdot \cos\left(\frac{\alpha}{2}\right) = 2\,\frac{kqq}{a^2}\,\frac{\sqrt{3}}{2} = k\frac{q^2}{a^2}\sqrt{3}$$

$$|\vec{F}_0'| = k\frac{qQ}{\left(\frac{2}{3}v\right)^2} = k\frac{qQ}{\frac{4}{9}v^2} = k\frac{9Qq}{4v^2}$$

$$-F_0 = F_0'$$

$$-k\frac{q^2}{a^2}\sqrt{3} = k\frac{9Qq}{4\left(\frac{3}{4}a^2\right)}$$

$$q\sqrt{3} = -\frac{9Q}{3} = -3Q$$

$$Q = -q\frac{\sqrt{3}}{3} = -q\frac{\sqrt{3}\sqrt{3}}{3\sqrt{3}} = \frac{-q}{\sqrt{3}}$$

$\boxed{7.19}$

$$\varphi = \frac{q}{4\pi\varepsilon r}$$

$$\left.\begin{aligned} \varphi_1 r_1 &= \frac{q}{4\pi\varepsilon} \\ \varphi_2 r_2 &= \frac{q}{4\pi\varepsilon} \end{aligned}\right\} \varphi_1 r_1 = \varphi_2 r_2$$

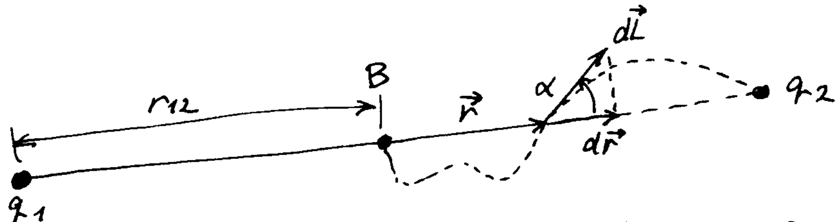
$$\varphi_2 = \frac{r_1}{r_2}\varphi_2 = \frac{2\cdot 10^{-2}}{0{,}05\cdot 10^{-2}}\cdot 10^4 = 4\cdot 10^5\,V = 400\,kV$$

## Poznámka k úlohe 7.17

Vyšetrime interakčnú energiu takejto sústavy nábojov. Potenciálna energia dvoch bodových nábojov môže byť určená nasledujúcim postupom. Približujme náboj $q_2$, ktorý je prakticky v nekonečne najkratšou cestou po priamke k náboju $q_1$ až sa náboj $q_2$ zastaví v bode B vo vzdialenosti $r_{12}$ od náboja $q_1$. Pritom sme vykonali prácu rovnú

$$W = -\int_{\infty}^{r_{12}} F_{21} \cdot dr = -\frac{q_1 q_2}{4\pi\varepsilon_0}\int_{\infty}^{r_{12}} \frac{dr}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} .$$

Keď budeme náboj $q_2$ približovať po ľubovoľnej dráhe L dostaneme




$$W = -\int_{(L)} \vec{F}_{21} \cdot d\vec{L} = -\frac{q_1 q_2}{4\pi\varepsilon_0}\int_{(L)} \frac{\vec{r} \cdot d\vec{L}}{r^3} = -\frac{q_1 q_2}{4\pi\varepsilon_0}\int_{(L)} \frac{|\vec{r}| \cdot |d\vec{L}| \cdot \cos\alpha}{r^3} =$$

$$= -\frac{q_1 q_2}{4\pi\varepsilon_0}\int_{\infty}^{r_{12}} \frac{dr}{r^2} = -\frac{q_1 q_2}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r_{12}} = -\frac{q_1 q_2}{4\pi\varepsilon_0}\left[-\frac{1}{r_{12}} + \emptyset\right] =$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} . \quad \left(\vec{r} \cdot d\vec{L} = |\vec{r}| \cdot |d\vec{L}| \cdot \cos\alpha = r \cdot dr\right)$$

Výsledná práca nezávisí od spôsobu približovania nábojov. Keď sa priblíži k tejto dvojici tretí náboj, podľa <u>princípu superpozície</u> bude výsledná potenciálna energia rovná

$$E_p = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right).$$

Všeobecne potenciálnu energiu sústavy N nábojov môžeme vyjadriť vzťahom

$$E_p = \frac{1}{8\pi\varepsilon_0}\sum_{i \neq j}^{N} \frac{q_i q_j}{r_{ij}} .$$

(Sčítavame cez rôzne i a j, čiže ~~sa~~ každá dvojica ~~sa~~ nábojov sa uvažuje dvakrát, preto sme pred znak sumy vybrali $\frac{1}{2}$.)
Výsledný stav potenciálnej energie sústavy elektrických nábojov môže byť kladný, záporný i nulový.

Pomocou všeobecného vzťahu pre pot. energiu sústavy N nábojov môžeme vyjadriť interakčnú ~~...~~ energiu <u>v našom zadaní</u>.
Táto sústava pozostáva z troch dvojíc nábojov $q\,q$ a $q\,Q$. Pre

takéto dvojice môžeme napísať

$$E_{qq}^{(1)} = \frac{qq}{4\pi\varepsilon_0 a} \quad , \quad E_{qQ}^{(1)} = -\frac{3qQ}{4\pi\varepsilon_0\sqrt{3}a} \quad .$$

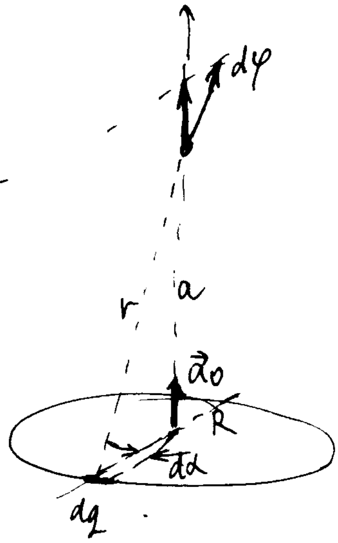
Pre celú sústavu nábojov potom platí vzťah

$$E = 3\cdot E_{qq}^{(1)} + 3\cdot E_{qQ}^{(1)} = \frac{3q^2}{4\pi\varepsilon_0 a} - \frac{9qQ}{\sqrt{3}\,4\pi\varepsilon_0 a} =$$

$$= \frac{3q^2}{4\pi\varepsilon_0 a} - \frac{3\sqrt{3}qQ}{4\pi\varepsilon_0 a} = \frac{3q}{4\pi\varepsilon_0 a}\left(q - \sqrt{3}Q\right).$$

Keď $q = \sqrt{3}Q$ je energia sústavy nulová. Akákoľvek zmena polohy jedného z nábojov vedie k vzniku nenulovej sily pôsobiacej na náboj $Q$, čím sa celý systém nábojov uvedie do nestabilného stavu! Z uvedeného príkladu vidíme, že rovnováha statických bodových nábojov nie je stabilná. Skutočne platí Earnshawova veta, že náboje nemôžeme udržiavať v stabilnej rovnováhe len elektrostatickými silami.

$\boxed{7.21}$




$$d\varphi = \frac{1}{4\pi\varepsilon}\frac{dq}{r}$$

$$r^2 = a^2 + R^2$$

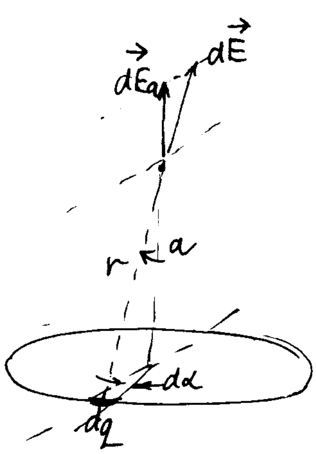
$$d\varphi = \frac{1}{4\pi\varepsilon}\frac{dq}{\sqrt{a^2+R^2}}$$

$$dq = \tau R\, d\alpha = \frac{q}{2\pi R} R\, d\alpha = q\frac{d\alpha}{2\pi}$$

$$\varphi = \frac{1}{4\pi\varepsilon\sqrt{a^2+R^2}}\int dq = \frac{q}{4\pi\varepsilon\sqrt{a^2+R^2}}\underbrace{\int_0^{2\pi}\frac{d\alpha}{2\pi}}_{1} =$$

$$= \frac{q}{4\pi\varepsilon\sqrt{a^2+R^2}}$$

$$\vec{E} = -\operatorname{grad}\varphi = -\frac{d\varphi}{da}\vec{a}_0 = -\vec{a}_0\frac{q}{4\pi\varepsilon}\frac{d}{da}\left(\frac{1}{\sqrt{a^2+R^2}}\right) = \frac{a\,q\,\vec{a}_0}{4\pi\varepsilon\,(a^2+R^2)^{3/2}}$$




$$|d\vec{E}_a| = |d\vec{E}|\cos\alpha$$

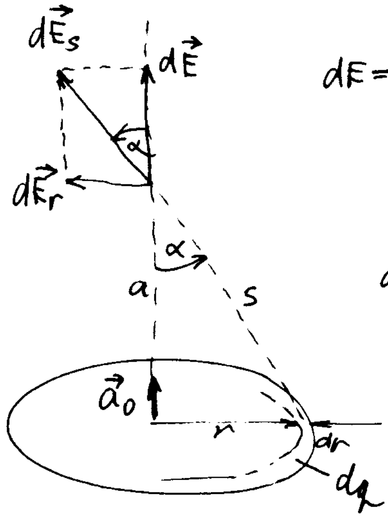
$$dE = \frac{1}{4\pi\varepsilon}\frac{dq}{r^2} = \frac{dq}{4\pi\varepsilon(a^2+R^2)}$$

$$\cos\alpha = \frac{a}{r} = \frac{a}{\sqrt{a^2+R^2}}$$

$$dE_a = \frac{a\,dq}{4\pi\varepsilon(a^2+R^2)(a^2+R^2)^{1/2}}$$

$$E_a = \frac{a\,q}{4\pi\varepsilon\,(a^2+R^2)^{3/2}}\underbrace{\int_0^{2\pi}\frac{d\alpha}{2\pi}}_{1} = \frac{a\,q}{4\pi\varepsilon\,(a^2+R^2)^{3/2}}$$

$\boxed{7.22}$




$$dE = dE_s \cos\alpha = \frac{dq}{4\pi\varepsilon s^2}\cos\alpha$$

$$r = a\,tg\alpha \; ; \; s = \frac{a}{\cos\alpha} \; ; \; \sigma = \frac{q}{\pi R^2}$$

$$dr = \frac{a\,d\alpha}{\cos^2\alpha} \qquad dq = 2\pi r \sigma dr$$

$$dE = \frac{2\pi\sigma}{4\pi\varepsilon}\, a\,tg\alpha\, \frac{a\,d\alpha}{\cos^2\alpha} \cdot \frac{a}{\frac{a^3}{\cos^3\alpha}} = \frac{2\pi\sigma}{4\pi\varepsilon}\, tg\alpha \cos\alpha\, d\alpha =$$

$$= \frac{\sigma}{2\varepsilon}\sin\alpha\, d\alpha$$

$$E = \frac{\sigma}{2\varepsilon}\int_0^{arctg\left(\frac{R}{a}\right)} \sin\alpha\, d\alpha = \frac{\sigma}{2\varepsilon}\Big[-\cos\alpha\Big]_0^{arctg\left(\frac{R}{a}\right)} =$$

$$= \frac{\sigma}{2\varepsilon}\left[1 - \cos\left(arctg\left(\frac{R}{a}\right)\right)\right] = \frac{\sigma}{2\varepsilon}\left[1 - \frac{1}{\sqrt{1 + tg^2\left(arctg\left(\frac{R}{a}\right)\right)}}\right] =$$

$$= \frac{\sigma}{2\varepsilon}\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R}{a}\right)^2}}\right] = \frac{\sigma}{2\varepsilon}\left[1 - \frac{a}{\sqrt{a^2+R^2}}\right] =$$

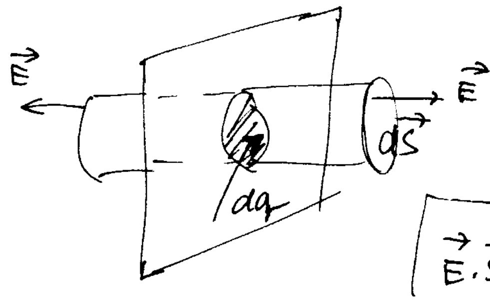
$$= \frac{q}{2\pi\varepsilon R^2}\left[1 - \frac{a}{\sqrt{a^2+R^2}}\right]$$

$$E \cdot \vec{a}_0 = \frac{q}{2\pi\varepsilon R^2}\left(1 - \frac{a}{\sqrt{a^2+R^2}}\right)\vec{a}_0$$

$$\underline{\vec{E} = \frac{q}{2\pi\varepsilon R^2}\left(1 - \frac{a}{\sqrt{a^2+R^2}}\right)\vec{a}_0}$$

$\boxed{7.23}$

a/



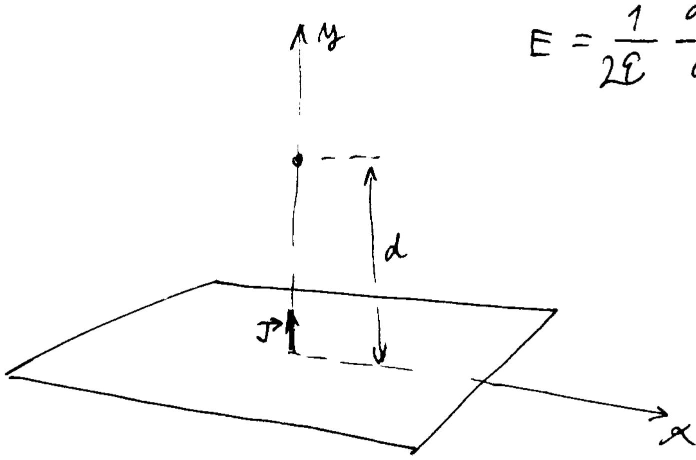
$$\boxed{\vec{E}.\vec{S} = \frac{q}{\varepsilon}}$$

$$\sigma = \frac{dq}{dS}$$

$$|\vec{E}| \cdot 2 |d\vec{S}| = \frac{dq}{\varepsilon}$$

$$E = \frac{1}{2\varepsilon} \frac{dq}{dS} = \frac{\sigma}{2\varepsilon}$$

b/



$$\vec{E} = -\operatorname{grad} \varphi = -\vec{J} \frac{d\varphi}{dy}$$

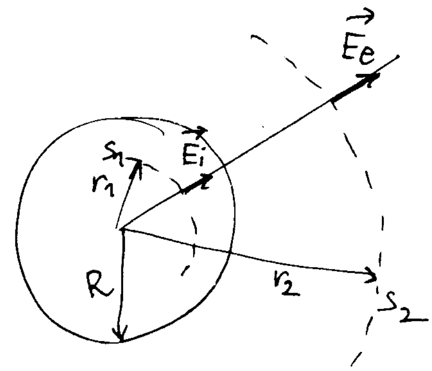
$$E\vec{J} = -\vec{J} \frac{d\varphi}{dy}$$

$$\Downarrow$$

$$d\varphi = -E . dy$$

$$\varphi = -E \int_0^d dy = -\frac{\sigma}{2\varepsilon} d$$

$\boxed{7.25}$




keď bude $\rho = \frac{3Q}{4\pi R^3} = \text{konšt.},$

potom:

a/ $0 \leq r \leq R$

$$q_i = \frac{4}{3}\pi r^3 \rho$$

$$E_i \cdot 4\pi r^2 = \frac{1}{\varepsilon}\,\frac{4}{3}\pi r^3 \rho$$

$$\boxed{E_i = \frac{\rho}{3\varepsilon} r} = \frac{q r}{4\pi\varepsilon R^3}$$

Pre $\rho = \emptyset \quad E_i = \emptyset$

$$\varphi_i = -\int_\infty^R E\,dr = -\frac{q}{4\pi\varepsilon}\int_\infty^R \frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_\infty^R = \frac{q}{4\pi\varepsilon R} = \frac{qR}{4\pi\varepsilon R^2} = \underline{\underline{\frac{\sigma R}{\varepsilon}}}$$

alebo

b/ $r > R \qquad q_e = \frac{4}{3}\pi R^3 \rho = q$

$$E_e \cdot 4\pi r^2 = \frac{1}{\varepsilon}\cdot\frac{4}{3}\pi R^3 \rho$$

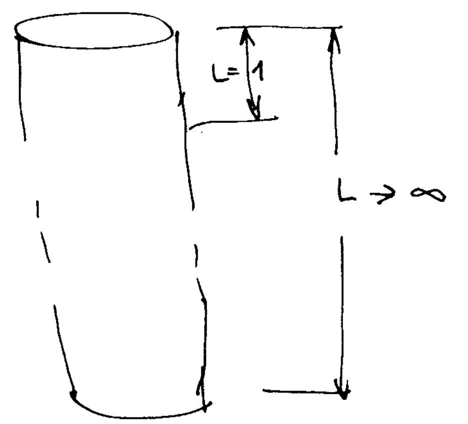
$$E_e = \frac{\rho R^3}{3\varepsilon r^2} = \underline{\underline{\frac{q}{4\pi\varepsilon r^2}}}$$

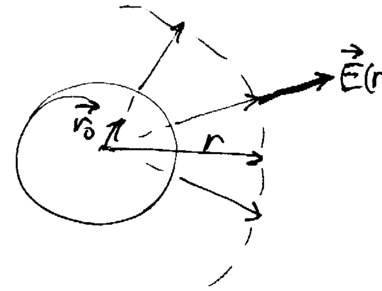
$$\varphi_e = -\int_r^R E\,dr = -\frac{q}{4\pi\varepsilon}\int_r^R \frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_r^R = \frac{q}{4\pi\varepsilon R} - \frac{q}{4\pi\varepsilon r} =$$

$$= \frac{\sigma R}{\varepsilon} - \frac{\sigma R^2}{\varepsilon r} = \underline{\underline{\varphi_i - \frac{\sigma R^2}{\varepsilon r}}}$$

Na povrchu gule je potenciál konštantný a teda vo vnútri gule je potenciál tiež konštantný, lebo intenzita poľa vo vnútri gule je rovná nule.

## 7.27



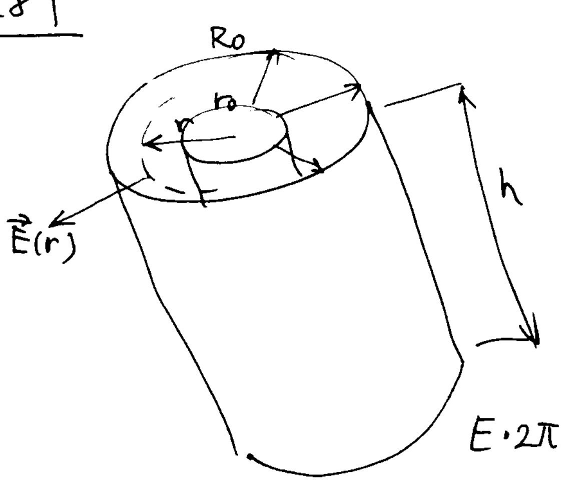

$$E \cdot 2\pi r L = \frac{q}{\varepsilon}$$

$$\vec{r}_0 \cdot E = \frac{q}{L} \frac{1}{2\pi \varepsilon r} \cdot \vec{r}_0$$

$$\vec{E} = \frac{1}{2\pi\varepsilon} \cdot \frac{q}{L} \vec{r}_0$$

$$L=1 \quad \underline{\vec{E} = \frac{q}{2\pi r \varepsilon} \vec{r}_0}$$

## 7.28

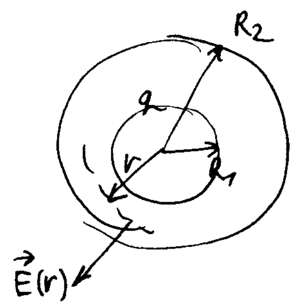



$$E \cdot 2\pi r h = \frac{q}{\varepsilon} \Rightarrow E = \frac{q}{2\pi h \varepsilon} \cdot \frac{1}{r}$$

$$\varphi = \int_{r_0}^{R_0} E\, dr = \frac{q}{2\pi h \varepsilon} \int_{r_0}^{R_0} \frac{dr}{r} = \frac{q}{2\pi \varepsilon h} \ln\left|\frac{R_0}{r_0}\right| \Rightarrow \frac{q}{2\pi h \varepsilon} = \frac{\varphi}{\ln\left|\frac{R_0}{r_0}\right|}$$

тогда $\boxed{E = \frac{\varphi}{r \ln\left|\frac{R_0}{r_0}\right|}}$

$\boxed{7.29}$




Kapacita vodiča:

$$C = \frac{q_{celk.}}{\varphi}$$

$$\varphi = \int E\,dr = \frac{q}{4\pi\varepsilon}\int_{R_1}^{R_2}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_{R_1}^{R_2} =$$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon} \Rightarrow E = \frac{q}{4\pi\varepsilon r^2}$$

$$= \frac{q}{4\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{q}{4\pi\varepsilon}\cdot\frac{R_2 - R_1}{R_1 R_2}$$

$q_{celk} = q$

$$C = \frac{q}{\frac{q}{4\pi\varepsilon}\cdot\frac{R_2-R_1}{R_1R_2}} = 4\pi\varepsilon\,\frac{R_1 R_2}{R_2 - R_1}$$