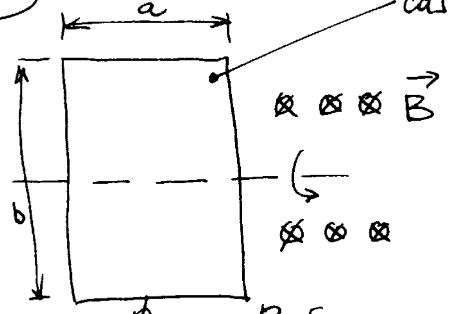
10.1



čas pol otáčky

$t = \frac{1}{2}T = \frac{1}{2}\frac{1}{f} = \frac{1}{2f}$

$\Delta\phi = \phi_{max} - (-\phi_{max}) = 2\phi_{max}$

$\phi_{max} = B \cdot S$

$$U_i = -\frac{\Delta\phi}{\Delta t} = -\frac{2\phi_{max}}{0-\frac{1}{2f}} = 4f\phi_{max} = 4fB\cdot S = 4fB\cdot a\cdot b =$$

$$= 4f\mu_0 H a\cdot b = 4\cdot\frac{1200}{60}\cdot 4\pi\cdot 10^{-7}\cdot \cancel{0{,}048}\cdot 0{,}048\cdot 10^{7}\cdot 0{,}25\cdot 0{,}3 =$$

$$= 3{,}61911\,V$$

- Iný spôsob: $\underbrace{t=0 \quad \phi = \phi_{max}}_{\Downarrow}$, $B = \text{konšt.}$

$$S = S_0\cos\omega t$$

$$U_i = -\frac{d\phi}{dt} = -\frac{d(BS)}{dt} = -B\frac{dS}{dt} = -BS_0\frac{d}{dt}(\cos\omega t) = \underbrace{BS_0\omega}_{U_0}\sin\omega t =$$

$$= U_0 \sin\omega t$$

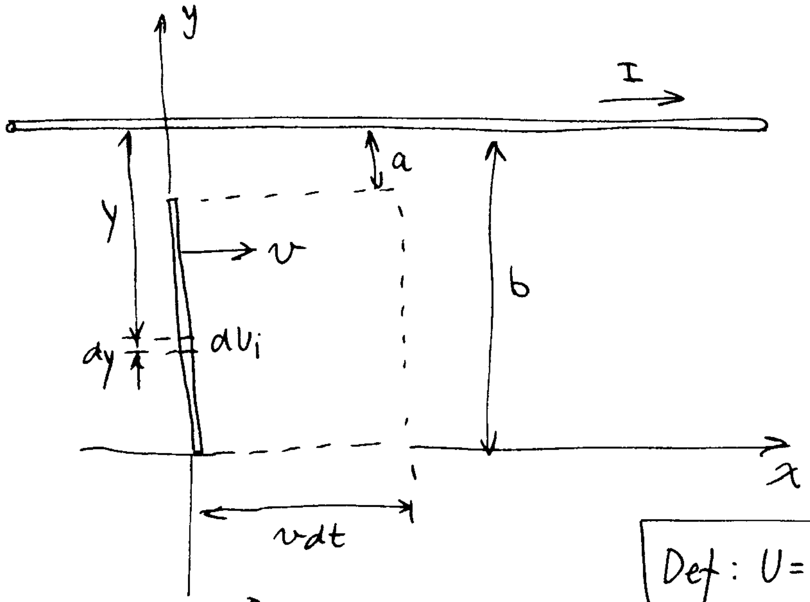
$$\boxed{\text{Časová stredná hodnota fyzikálnej veličiny} \quad \langle x\rangle = \frac{1}{T}\int_0^T x(t)\,dt}$$

$$\langle U\rangle = \frac{1}{\frac{T}{2}}\int_0^{\frac{T}{2}} U_0\sin\omega t\,dt = \frac{2U_0}{T}\int_0^{\frac{T}{2}}\sin\omega t\,dt = \frac{2U_0}{T}\left[-\cos\omega t\right]_0^{\frac{T}{2}} =$$

$$= \frac{2U_0}{T\cdot\omega}\left[\cos\left(\frac{2\pi}{T}\cdot\frac{T}{2}\right) + \cos 0\right] = \frac{2U_0}{\frac{2\pi}{\omega}\cdot\omega}\left[1+1\right] = \frac{2U_0}{\pi} =$$

$$= \frac{2\cdot B\cdot S_0\omega}{\pi} = 4BS_0 f = 4f\mu_0 H a\cdot b$$

10.4



Def: $U = BLv$

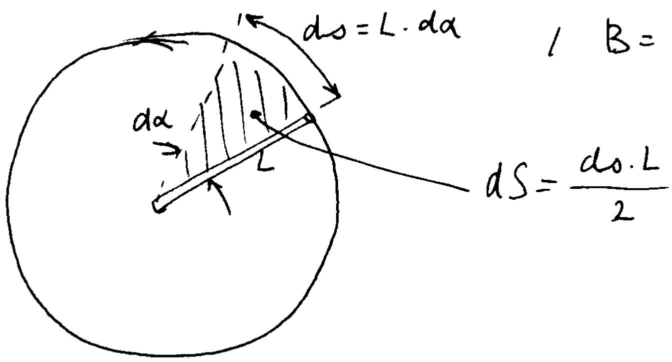
$$B = \frac{\mu_0 I}{2\pi y} \quad , \quad dU_i = B v\, dy$$

$$dU_i = B v\, dy = \frac{\mu_0 I}{2\pi} v \frac{dy}{y}$$

Pre celú tyč: $U_i = \frac{\mu_0 I}{2\pi} v \int_a^b \frac{dy}{y} = \frac{\mu_0 I}{2\pi} v \ln\left|\frac{b}{a}\right| =$

$$= \frac{4\pi \cdot 10^{-7} \cdot 40}{2\pi} \cdot 2 \ln\left|\frac{100}{10}\right| = \underline{3{,}684\,136 \cdot 10^{-5}\,V}$$

10.5



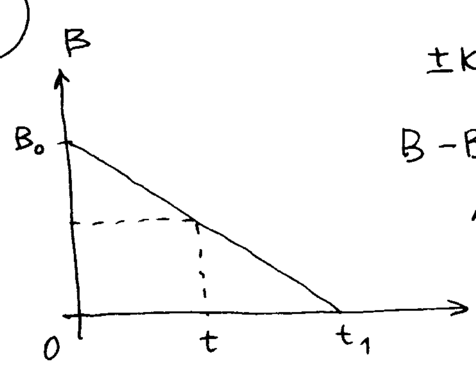
/ B = konšt.

$$U_i = \left|-\frac{d\phi}{dt}\right| = B\frac{dS}{dt} = B\frac{L}{2}\frac{d\sigma}{dt} = B\frac{L^2}{2}\frac{d\alpha}{dt} = B\frac{L^2}{2}\omega =$$

$$= \frac{BL^2}{2} 2\pi f = \pi B L^2 f$$

$$f = \frac{U_i}{\pi B L^2} = \frac{0{,}628}{\pi \cdot 0{,}2 (10 \cdot 10^{-2})^2} = 99{,}949\,302\, s^{-1} \approx \underline{\underline{100\, s^{-1}}}$$

10.6



$$\pm k = \frac{B-B_0}{t-t_0}$$

$$B - B_0 = \pm k(t-t_0)$$

$$\boxed{B = B_0 \pm k\,\Delta t}$$

$$k = \frac{B_1 - B_0}{t_1 - t_0} = \frac{\emptyset - B_0}{t_1 - \emptyset} = -\frac{B_0}{t_1}$$

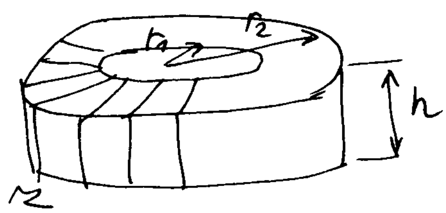
$$B = B_0 - B_0\frac{t}{t_1}$$

$$B = B_0\left(1 - \frac{t}{t_1}\right)$$

$$U_i = -\frac{\Delta\phi}{\Delta t} = -\frac{\Delta B . S}{\Delta t} = -\frac{(B-B_0)\pi R^2}{t_1 - t_0} =$$

$$= \frac{\emptyset - B_0 \pi R^2}{t_1 - \emptyset} = \underline{\underline{\frac{B_0 \pi R^2}{t_1}}}$$

10.15



Na výpočet použijeme výsledok z úlohy 9.23

$$\phi = \frac{\mu_0 I N}{2\pi} h \ln\left|\frac{r_2}{r_1}\right| .$$

Indukčnosť vodiča je definovaná ako:

$$L = \frac{\phi_{celk}}{I} .$$

$$\phi_{celk} = N\phi \,,\qquad L = \frac{N\phi}{I} = \underline{\underline{\frac{\mu_0 N^2 h}{2\pi} \ln\left|\frac{r_2}{r_1}\right|}}$$

10.16

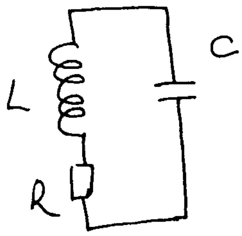
$\phi_M = N\phi$ $\quad$ ϕ_M je vzájomný indukčný tok dvoch vinutí

$$M = \frac{\phi_{M12}}{I_1} = \frac{\phi_{M21}}{I_2}$$

$$\phi_{M12} = N_2 \phi_1 = N_2 \frac{\mu_0 N_1 I_1}{2\pi} h \ln\left|\frac{r_2}{r_1}\right|$$

$$M = \underline{\underline{\frac{\mu_0 N_1 N_2}{2\pi} h \ln\left|\frac{r_2}{r_1}\right|}}$$

10.20



Nech $R = 0$

$$U_C + U_L = 0$$

$$U_L = L\frac{dI}{dt}, \quad U_C = \frac{q}{C} = U$$

$$dq = C\,dU$$

$$I\,dt = C\,dU$$

$$I = C\frac{dU}{dt}$$

$$\frac{I}{C} = \frac{dU}{dt}$$

$$\boxed{U + L\frac{dI}{dt} = 0} \quad (1)$$

$$L\frac{d^2I}{dt^2} + \frac{dU}{dt} = 0$$

$$\boxed{\frac{d^2I}{dt^2} + \frac{I}{LC} = 0} \quad (2)$$

Z dôvodu prehľadnosti budeme ďalej časovú deriváciu označovať bodkou nad veličinou. Rovnicu (2) vynásobíme $\dot I$ a pri riešení použijeme nasledujúce identity

$$I\dot I \equiv \frac{d}{dt}\left(\frac{1}{2}I^2\right),$$

$$\dot I\ddot I \equiv \frac{d}{dt}\left(\frac{1}{2}\dot I^2\right).$$

$$\dot I\ddot I + \frac{1}{LC}I\dot I = 0,$$

$$\frac{d}{dt}\left(\frac{1}{2}\dot I^2 + \frac{1}{LC}\frac{1}{2}I^2\right) = 0.$$

$\dot I^2 + \frac{1}{LC}I^2$ je kladná konštanta. Označme túto konštantu ako $\frac{A^2}{LC}$, potom

$$\dot I^2 + \frac{1}{LC}I^2 = \frac{A^2}{LC}$$

$$\dot I^2 = \frac{1}{LC}(A^2 - I^2)$$

$$\dot I = \frac{1}{\sqrt{LC}}\sqrt{A^2 - I^2}$$

$$\frac{dI}{dt} = \frac{1}{\sqrt{LC}}\sqrt{A^2 - I^2}$$

$$\frac{dI}{\sqrt{A^2 - I^2}} = \frac{dt}{\sqrt{LC}}.$$

Túto rovnicu riešime integráciou. Použijeme substitučnú metódu: $I = A\sin x$, čiže $dI = A\cos x\,dx$. Po dosadení dostávame

$$\frac{A\cos x\,dx}{\sqrt{A^2-A^2\sin^2 x}} = \frac{dt}{\sqrt{LC}}$$

$$\frac{A\cos x\,dx}{A\sqrt{1-\sin^2 x}} = dx = \frac{dt}{\sqrt{LC}}$$. Po integrácii dostaneme $x = \frac{t}{\sqrt{LC}} + \alpha$ a po spätnom dosadení

$$I = A\sin\left(\frac{t}{\sqrt{LC}}+\alpha\right) =$$
$$= I_0 \sin(\omega t + \alpha),$$

kde $\omega = \frac{1}{\sqrt{LC}}$, I_0 a α sú ľubovoľné integračné konštanty. V časovom okamihu $t=0$ je $I=0$, teda

$$I_0 \sin\alpha = 0$$
$$\sin\alpha = 0$$
$$\alpha = 0$$

Rovnicu (1) rozkladaním Lenzovho zákona môžeme zapísať v tvare $U - L\frac{dI}{dt} = 0$.

V čase $t=0$ $U=U_0$, $\frac{dI}{dt} = I_0\omega \underbrace{\cos\omega t}_{1} = I_0\omega$

$$U_0 - L I_0 \omega = 0$$
$$I_0 = \frac{U_0}{L\omega} = \frac{U_0\sqrt{LC}}{L} =$$

$$= U_0\sqrt{\frac{C}{L}} = U_0\sqrt{\frac{\varepsilon S}{d\cdot L}} = 100\sqrt{\frac{2\cdot 8{,}854\cdot 10^{-12}\cdot 0{,}45}{0{,}1\cdot 10^{-3}\cdot 0{,}07}} =$$

$$= 1{,}066\,945\cdot 10^{-5}\,A \approx 0{,}107\,A$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{2\pi}{T}$$

$$T = 2\pi\sqrt{LC} = 2\pi\sqrt{L\cdot\frac{\varepsilon S}{d}} = 2\pi\sqrt{0{,}07\cdot\frac{2\cdot 8{,}854\cdot 10^{-12}\cdot 0{,}45}{0{,}1\cdot 10^{-3}}} =$$

$$= 4{,}692\,668\cdot 10^{-4}\,s \approx 0{,}47\,ms$$

Iný spôsob riešenia. Napíšme rovnicu (2) v operátorovom tvare

$$(D^2+\omega^2)I = 0, \quad \text{kde } \omega^2 = \frac{1}{LC}$$

$$D^2+\omega^2 = 0$$

$$D^2 = -\omega^2$$

$$D = \pm\sqrt{-\omega^2} = \pm\omega\sqrt{-1} = \pm i\omega.$$

Všeobecné riešenie hľadáme v tvare

$$I = Ae^{i\omega t} + Be^{-i\omega t} = A(\cos\omega t + i\sin\omega t) + B(\cos\omega t - i\sin\omega t) =$$

$$= A\cos\omega t + iA\sin\omega t + B\cos\omega t - iB\sin\omega t = (A+B)\cos\omega t +$$

$$+ i(A-B)\sin\omega t.$$

Nech $A+B = C\sin\alpha$ a $i(A-B) = C\cos\alpha$, potom

$$I = C\sin\alpha\cos\omega t + C\cos\alpha\sin\omega t = C(\sin\alpha\cos\omega t + \cos\alpha\sin\omega t) =$$

$$\boxed{\sin(X+Y) = \sin X\cos Y + \cos X\sin Y}$$

$$= C\sin(\omega t+\alpha) = \underline{\underline{I_0\sin(\omega t+\alpha)}}.$$

Ďalej postupujeme ako pri predchádzajúcom spôsobe riešenia.

10.21

$$E_m = \frac{1}{2}LI^2, \qquad E_e = \frac{1}{2}CU^2$$

$$I = I_0 \sin(\omega t + \alpha)$$

V časovom okamihu $t=0$ nech $I=0$, potom

$$I_0 \sin\alpha = 0$$
$$\sin\alpha = 0$$
$$\alpha = 0, \text{ čiže}$$

$$I = I_0 \sin\omega t,$$
$$U = U_0 \cos\omega t.$$

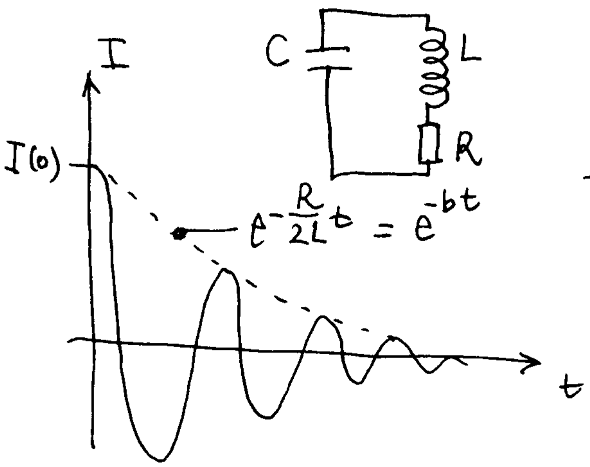
$$\boxed{\frac{1}{2}LI_0^2 = \frac{1}{2}CU_0^2}$$

zákon zachovania energie

$$\frac{E_m}{E_e} = \frac{\frac{1}{2}LI_0^2 \sin^2\omega t}{\frac{1}{2}CU_0^2\cos^2\omega t} = \cot g^2 \omega t = \cot g^2\left(\frac{2\pi}{T}\cdot\frac{T}{8}\right) =$$

$$= \cot g^2\left(\frac{\pi}{4}\right) = 1.$$

10.22



$$\lambda = \frac{I(t)}{I(t+T)} = \frac{I_0 e^{-bt}\cos\omega t}{I_0 e^{-b(t+T)}\cos[\omega(t+T)]} =$$

$$= \frac{I_0 e^{-bt}\cos\omega t}{I_0 e^{-bt}\cos[\omega(t+T)]\cdot e^{-bT}} = e^{bT}$$

$$\cos[\omega(t+T)] = \cos[\omega(t+\tfrac{2\pi}{\omega})] = \cos(\omega t + 2\pi) = \cos\omega t$$

$$\lambda = \frac{I(0)}{I(t)} = e^{bt}, \qquad I(t) = \frac{I(0)}{3}$$

$$\frac{I(0)}{\frac{I(0)}{3}} = e^{bt}$$

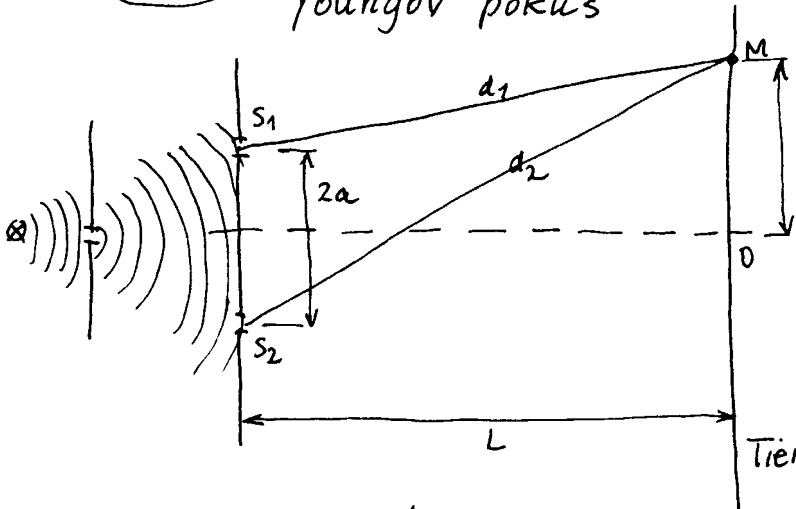
$$3 = e^{bt}$$

$$t = \frac{\ln 3}{b} = \frac{\ln 3}{\frac{R}{2L}} = \frac{2L}{R}\ln 3$$

$$t = \frac{2\cdot 5\cdot 10^{-3}}{9{,}7}\ln 3 = 1{,}133\cdot 10^{-3}\,s \approx 1\,ms$$

10.29

Youngov pokus



V roku 1801 Thomas Young experimentálne dokázal, že svetlo má vlastnosti vlnenia. Väčšina fyzikov v tej dobe pokladala svetlo za prúd častíc. Svojím pokusom demonštroval, že svetlo interferuje tak isto, ako zvukové, „vodné" a iné typy vĺn. Dokázal zmerať strednú vlnovú dĺžku slnečného svetla – 570 nm, ktorá sa málo líši od dnes prijatej hodnoty 555 nm.

$L \gg a, y$

$$d_1 = \sqrt{L^2 + (y-a)^2} = L\left[1 + \left(\frac{y-a}{L}\right)^2\right]^{1/2}$$

$$d_2 = \sqrt{L^2 + (y+a)^2} = L\left[1 + \left(\frac{y+a}{L}\right)^2\right]^{1/2}$$

Na úpravu použijeme pravidlo na rozvoj mnohočlena do binomického radu

$$(a+b)^p = \binom{p}{0}a^p + \binom{p}{1}a^{p-1}b + \binom{p}{2}a^{p-2}b^2 + \dots + \binom{p}{p-1}ab^{p-1} + \binom{p}{p}b^p$$

Keďže $L \gg a, y$ potom $\left|\frac{y+a}{L}\right| < 1$.

$$(1 \pm x)^p = 1 \pm \binom{p}{1}x + \binom{p}{2}x^2 \pm \dots + (-1)^p\binom{p}{p}x^p,$$ ~~kde~~

$$\binom{p}{k} = \frac{p!}{(p-k)!\,k!}, \quad \binom{p}{0} = 1, \quad \binom{p}{1} = p.$$

Keď $\left|\frac{y+a}{L}\right| < 1$ potom $\left(\frac{y+a}{L}\right)^2 \ll 1$, čiže

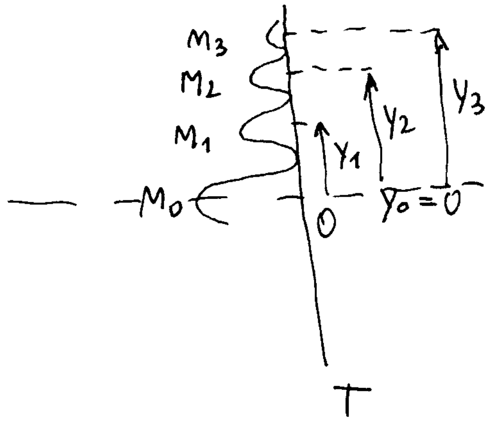
$$(1 \pm x)^{1/2} = 1 \pm \binom{\frac{1}{2}}{1}x \pm \dots \approx 1 \pm \frac{x}{2}.$$

$$d_1 \approx L\left[1 + \frac{1}{2}\left(\frac{y-a}{L}\right)^2\right] \quad a \quad d_2 \approx L\left[1 + \frac{1}{2}\left(\frac{y+a}{L}\right)^2\right].$$

Pre maximá interferujúcich lúčov musí byť splnená podmienka $\boxed{d_2 - d_1 = m\lambda}$, kde $m = 0, 1, 2, \dots$ Dráhový rozdiel je celočíselným násobkom vlnovej dĺžky použitého svetla.

$$d_2 - d_1 = L\left[1 + \frac{1}{2}\left(\frac{y+a}{L}\right)^2\right] - L\left[1 + \frac{1}{2}\left(\frac{y-a}{L}\right)^2\right] = \frac{(y+a)^2 - (y-a)^2}{2L} =$$

$$= \frac{y^2+2ay+a^2-(y^2-2ay+a^2)}{2L} = \frac{4ay}{2L} = \frac{2ay}{L} = m\lambda$$



Zo zadania úlohy vyplýva

$$y_1 - y_0 = y_2 - y_1 = y_3 - y_2 = \dots = \Delta y .$$

Napr. $y_1 - y_0 = \Delta y$ 1. maximum

$y_2 - y_1 = \Delta y$ 2. max.

$y_3 - y_2 = \Delta y$ 3. max.

$$(y_1 - y_0) + (y_2 - y_1) + (y_3 - y_2) = 3\Delta y$$

$$y_3 - y_0 = 3\Delta y$$

$$m = 3$$

Pre 2. maximum bude

$y_2 - y_0 = 2\Delta y$, teda $m = 2$.

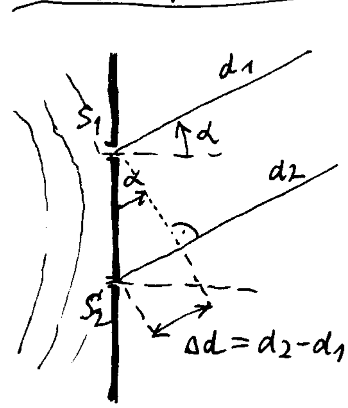
Číselný výpočet prevedme pre $m = 1$

$$\lambda = \frac{2ay}{m \cdot L} = \frac{d \cdot y}{m \cdot L} = \frac{0{,}6 \cdot 10^{-3} \cdot 10^{-3}}{1 \cdot 1} = 0{,}6 \cdot 10^{-6}\,m = \underline{\underline{600\,nm}},$$

pričom sme zaviedli označenie vzdialenosti štrbín $d = 2a$.

Iný spôsob:

Pre $L \gg 2a$ môžeme d_1 a d_2 považovať približne za rovnobežné lúče, ktoré sa šíria pod uhlom α vzhľadom k osi O (viď obrázok na predchádzajúcej strane a vľavo)



$$\Delta d = d_2 - d_1 = 2a \sin\alpha$$

Pre maximá svetelných prúžkov:

$$\Delta d = 2a \sin\alpha = (\text{celé číslo}) \cdot (\lambda)$$

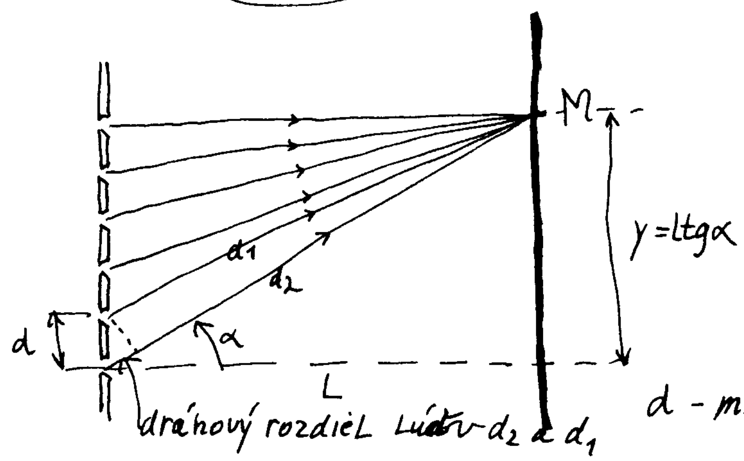
$$\boxed{2a \sin\alpha = m\lambda}, \quad m = 0, 1, 2, \dots$$

Pre minimá svetelných prúžkov:

$$\Delta d = 2a \sin\alpha = (\text{nepárne číslo}) \cdot (\tfrac{1}{2}\lambda)$$

$$\boxed{2a \sin\alpha = (m + \tfrac{1}{2})\lambda}, \quad m = 0, 1, 2, \dots$$

10.30



d - mriežková konštanta

V mieste M je svetlá čiara (maximum), keď dráhový rozdiel susedných lúčov je celočíselným násobkom vlnovej dĺžky použitého svetla.

$$d_2 - d_1 = d \cdot \sin\alpha = m\lambda, \quad m = 0, 1, 2, \dots$$

$$\sin\alpha = \frac{m\lambda}{d} \Rightarrow \alpha = \arcsin\left(\frac{m\lambda}{d}\right)$$

Pre prvý svetlý pásik

$$y_1 = L\,tg\,\alpha = L\,tg\left(\arcsin\left(\frac{m\lambda}{d}\right)\right) =$$

$$= 1 \cdot tg\left(\arcsin\left(\frac{1 \cdot 700 \cdot 10^{-9}}{\frac{10^{-3}}{100}}\right)\right) = 7{,}017\,213 \cdot 10^{-2}\,m$$

Pre tretí svetlý pásik

$$y_3 = 1 \cdot tg\left(\arcsin\left(\frac{3 \cdot 700 \cdot 10^{-9}}{\frac{10^{-3}}{100}}\right)\right) = 0{,}214\,789\,m$$

$$\Delta y = y_3 - y_1 = 0{,}144\,617\,m \approx \underline{\underline{14\,cm}}$$

10.31

$$d \cdot \sin\alpha = m\lambda$$

$$m = \frac{d \cdot \sin\alpha}{\lambda}$$

Podľa zadania úlohy pre pozorovanie červenej čiary v najvyššom ráde spektra musí byť splnená podmienka $\sin\alpha = 1$, čiže platí

$$m \leq \frac{d}{\lambda}$$

$$m \leq \frac{\frac{10^{-3}}{300}}{700 \cdot 10^{-9}} = \frac{10^{-3}}{300 \cdot 700 \cdot 10^{-9}} = 4{,}761\,905$$

$$m = 4$$

10.35

$$d \cdot \sin\alpha = m\lambda_r \Rightarrow m = \frac{d \cdot \sin\alpha}{\lambda_r}$$

$$d \cdot \sin\alpha = (m+1)\lambda_f$$

$$d \cdot \sin\alpha = \frac{d \cdot \sin\alpha}{\lambda_r} \cdot \lambda_f + \lambda_f$$

$$d \cdot \sin\alpha \left(1 - \frac{\lambda_f}{\lambda_r}\right) = \lambda_f$$

$$\sin\alpha = \frac{\lambda_f}{d\left(1 - \frac{\lambda_f}{\lambda_r}\right)}$$

$$\alpha = \arcsin\left(\frac{\lambda_f}{d \cdot \left(1 - \frac{\lambda_f}{\lambda_r}\right)}\right) = \arcsin\left(\frac{405 \cdot 10^{-9}}{\frac{10^{-3}}{310}\left(1 - \frac{405}{540}\right)}\right) =$$

$$= \arcsin(0{,}50625) = 30{,}414\,366^{\circ}$$